Yu. M. Veshkurtsev

THE FOUNDATIONS OF THE THEORY OF CONSTRUCTION OF NEW-GENERATION MODEMS

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The monography presents the fundamentals of the theory of construction new-generation modems. Modems are built on the principles of statistical communication theory, based on the use of a random signal (chaos) as a carrier of information. In such a signal, a characteristic function is modulated, which is a fundamental characteristic of a random process. The signal modulation and demodulation method is patented and allows you to create modems with efficiency and noise immunity indicators several orders of magnitude higher than those of the known devices of the same name. New-generation modems immediately improve the technical characteristics of digital IT equipment by several orders of magnitude, since they work without errors in wired and radio channels when receiving one hundred duodecillion of binary symbols.

The book is recommended for scientists and specialists in the field of digital communication systems, statistical radio engineering and instrumentation, and may be useful for graduate students, masters and students of relevant specialties.

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Introduction

Large-scale digitalization of our planet, artificial intelligence, and much more are coming. The level of people's comfort will be enhanced due to intelligent digital technologies based on modern digital systems. Each such system has many important characteristics, of which we single out noise immunity. It determines the number of errors in the document depending on the signal level and interference. Currently, digital systems provide one error per ten million bits of information. In relation to the document, we can talk about one error in the text with font 12 on one hundred sheets of A4 paper. Is it a lot or a little? If we consider a hundred pages of an e-book, then this is not enough. If we take a document in the form of one hundred receipts for the transfer of funds to recipients, then this is a lot, since an error can alter the amount of payment to any one of the hundred clients. There should not be a similar and any other mistake in the digital economy. Therefore, for the introduction of the digital economy, it will be necessary to increase the noise immunity of digital systems by several orders of magnitude at once.

Historically, it has become a rule for the transmission of messages to modulate the parameters of a deterministic oscillation, believing that its presence in practice is possible by inertia. This confidence existed until the results of measurements of amplitude, phase, and frequency fluctuations of physical sources of harmonic oscillations appeared in 1950. After that, it turned out that the deterministic vibration is nothing more than some kind of mathematical abstraction, which is unrealizable in practice. Then the experimentally established fluctuations of the oscillation parameters were hidden by the term "parasitic amplitude, phase, frequency fluctuations". This has survived to the present, they are being fought. And as a result of this, the theoretically established indicators of noise immunity of communication systems are still unattainable. To overcome the current situation and increase the noise immunity of digital systems by a big leap forward, we propose an alternative option, namely, to abandon the unpromising deterministic oscillation and switch to a random or, at the first stage, to a quasi-deterministic signal. Both signals, in our understanding, are synonymous with dynamic chaos, which opens up great opportunities in the development of new methods for transmitting, storing and processing information. When applying dynamic chaos, it would be appropriate to talk about digital systems with high noise immunity, in which the error probability will be $1 \cdot 10^{-41}$, i.e. one error when receiving one hundred duodecillion binary characters.

In addition, to achieve high noise immunity of digital systems, a new approach to signal modulation is proposed, which includes what was said above about chaos and the transition to modulation of the characteristic function of the signal, which, will serve as a “space suit” for the modulated signal by analogy with astronautics. The new method of random signal modulation will be conditionally called statistical modulation, which forms the basis of the theory of statistical communication. The results of the analysis show that with the help of the new method it is possible to achieve the limiting values of noise immunity, spectral and energy efficiency of digital systems.

Thus, the monography is devoted to increasing the noise immunity of digital data transmission systems by several orders of magnitude using new methods, techniques and devices. Its material complements the content of the avant-garde direction of statistical radio engineering, aimed at involving random processes in solving problems in the theory of statistical communication, which remain relevant to this day. Using random signals with probabilistic characteristics, the author in his research proves the promise of using dynamic chaos in a new generation of radio engineering devices, for example, in modems.

The monography considers various modem structures, algorithms for modulation and demodulation of a quasi-deterministic signal, and characteristic functions of signals that are proposed to be used in digital data transmission. Along the way, new knowledge was obtained regarding the characteristic function and the distribution law of signals, which were not known in probability theory. It is shown that the signal, characteristic function, signal modulation and demodulation algorithms in aggregate are the product of digital technology. The noise immunity of modems of the new generation when operating in a noisy channel was assessed qualitatively and quantitatively. At the same time, it was found that with a signal-to-noise ratio of 3 dB, the noise immunity of the modem is at least ten orders of magnitude higher than the same characteristic of known analogues, for example, with phase modulation.

1. CHARACTERISTIC FUNCTION

The characteristic function (ch.f.) is a probabilistic characteristic of a random process, or a random variable. It was proposed and used (1901) by the mathematician A.M. Lyapunov to prove the probability limit theorem. Since then, ch.f. has acquired independent significance and is successfully used to solve both fundamental and applied problems [1, 2]. In our opinion, it is appropriate to call it the characteristic function of A.M. Lyapunov, as it is done, for example, with the functions of F. Bessel, P. Dirac, P. Struve, O. Heaviside and other scientists. For more than fifty years, the characteristic function of A. Lyapunov served as a tool of mathematics and remains so to this day. Thanks to this function, mathematicians solve complex problems and construct difficult proofs of theorems. Since 1954 this function has been studied and applied by applied science. With its help, new results have been obtained in non-destructive testing and diagnostics, in communication technology, in noise filtering in the probability space, in detecting signals that are an order of magnitude or more superior to those previously known [2,3].

1.1. Characteristic function: definition, properties

A one-dimensional characteristic function is a statistical mean of an exponent with an imaginary exponent of the form $jV_m\xi\ (t)$, in which the random process ξ(t) is multiplied by an arbitrary real parameter V_m. Mathematical model ch. f. is represented by the expression

$$\Theta_1(V_m) = m_1\{\exp[jV_m\xi(t)]\}, \tag{1.1}$$

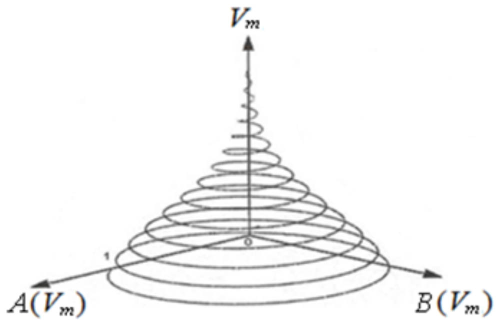
Where $\theta_1\ (V_m\ )$ is the characteristic function (ch.f.); $m_1\{\cdot\}$ is mathematical sign of the statistical average (expectation operator); $V_m = m\Delta V$ is the ch.f. parameter; ΔV is the discretization step (quantization) of the ch.f. parameter; m$\in$[-∞,+∞]. In expression (1.1), the number 1 denotes a one-dimensional function. Obviously, using the Euler's formula, we can write

$$\begin{aligned}\theta_1(V_m) &= m_1\{\cos[V_m\xi(\mathrm{t})] + j\sin[V_m\xi(\mathrm{t})]\} = \\ &= A(V_m) + jB(V_m),\end{aligned} \tag{1.2}$$

where $A(V_m\ ) = \mathrm{m}_1\{\cos[V_m\xi(\mathrm{t})]\}$ is the real part of ch.f.; $B(V_m\ ) = m_1\{\sin[V_m\xi(\mathrm{t})]\}$ is the imaginary part of the ch.f. Then there will be an equality

$$\theta_1(V_m) = |\theta_1\ (V_m)|\ \exp\ [j\Upsilon(V_m)]. \tag{1.3}$$

Here $|\theta_1(V_m)| = \sqrt{A^2(V_m) + B^2(V_m)}$, $Y(V_m) = arctg[B(V_m)/A(V_m)]$ are the modulus and argument of ch.f., respectively. Geometric interpretation ch. f. shown in figure 1.1. It depicts a spatial figure formed by the rotation of the radius - a vector with a length equal to the module $|\theta_1(V_m)|$, around the axis on which the values of the parameter V_m are plotted. In this case, the projections of a point belonging to the figure on the coordinate axes are respectively equal to the real and imaginary parts of ch. f. The shape and dimensions of the figure are determined by the random process $\xi(t)$. The figure in picture 1.1 is built for a Gaussian process with $m_1 = 0.3$, $\sigma = 0.01$ and looks like a funnel, where m_1 is the expectation of the process; σ is the mean square deviation (MSD) of the process. In the section of the figure, for a fixed value of ξV_m, a circle is obtained.



Picture 1.1. *One of the possible geometric interpretations of the ch.f.*

From a physical point of view, the parameter V_m is the coefficient (or multiplicity) of amplification (weakening) of the instantaneous values of the random process, and the product $V_m\,\xi(t)$ is the instantaneous phase of the analytical signal

$$w(t) = e^{jV_m\xi(t)} = \cos[V_m\xi(t)] + j\sin[V_m\xi(t)]\,. \qquad (1.4)$$

Then ch.f. is the mathematical expectation of an analytical signal with a constant modulus $\sqrt{\cos^2[V_m\xi(t)] + \sin^2[V_m\xi(t)]} = 1$, while the random process $\xi(t)$ only determines the law of change of the instantaneous phase of the signal (1.4). In this case, the meaning of the picture in Figure 1.1 is as follows. With an increase in the multiplication factor of the instantaneous phase of the analytical signal, its expectation decreases, since the rapidly changing signal is strongly averaged, as a result of which its constant component tends to zero. For a certain multiplication factor V_m, it even becomes equal to zero. In this place, the top of the spatial figure appears.

The well-known relation of the ch.f. with probabilistic characteristics of a random process is represented by the formula [4]

$$\theta_1(V_m) = \int_{-\infty}^{+\infty} W_1(x)e^{jV_m x}dx\,, \qquad (1.5)$$

where $W_1(x)$ is the one-dimensional probability density of the random process $\xi(t)$. In a particular case, we have

$$\theta_1(V_m) = \int_{-\infty}^{+\infty} W_1(x)\cos(V_m x)\,dx\,, \tag{1.6}$$

if the imaginary part of the ch.f. equals zero. This implies that if ch. f. takes on only real values, then it is even, and the corresponding probability density will be symmetric. Conversely, if the probability density of a random process is symmetric, then ch. f. is a real, even function of the parameter V_m. In addition to (1.5), a relation x is established ch. f. with initial and central moment functions of a random process. The book [4] gives the equality

$$\left(\frac{d^k\Theta_1(V_m)}{dV_m^{(k)}}\right)_{V_m=0} = j^k m_k\{\xi(t)\}. \tag{1.7}$$

It follows from this that the initial moment functions of the kth order differ from the value of the *k*-x derivatives of ch. f. if $V_m = 0$ only by the factor j^k. In a particular case, if $k = 1$, we obtain an expression for the expectation of a random process

$$m_1\{\xi(\text{t})\} = -j\dot{\theta}_1(V_m). \tag{1.8}$$

In the general case, if there are initial moments of any order, then, as follows from (1.7), ch. f. can be represented next to Maclaurin

$$\Theta_1(V_m) = 1 + \sum_{\text{k}=1}^{\infty}[m_k\{\xi(\text{t})\}/\text{k}!](jV_m)^{\text{k}}. \tag{1.9}$$

If we expand not ch.f. into a Maclaurin series, but a cumulant function as ln $\theta_1(V_m)$, then we obtain the expression

$$M(V_m) = \ln\theta_1(V_m) = \sum_{k=1}^{\infty}[\chi_k/\text{k}!](jV_m)^k. \tag{1.10}$$

The coefficients of this series, called cumulants (or semi-invariants) of the distribution, are expressed in terms of central moments by the formulas [4]

$$\chi_1 = m_1\{\xi(\text{t})\},\quad \chi_2 = M_2\{\xi(\text{t})\},\quad \chi_3 = M_3,$$
$$\chi_4 = M_4 - 3\text{M}_2^2,\quad \chi_5 = M_5 - 10M_2M_3, \tag{1.11}$$

where $M_k\{\xi(\text{t})\}$ is the central moment functions of the k^{th} order of the random process ξ(t). It can be seen from formulas (1.11) that the 1st order semi-invariant is equal to the expectation, and the 2nd order semi-invariant is equal to the variance of the random process.

For many random processes, the ch.f. is defined and calculated. Information about them is systematized in the literature [2,5]. A graph of the ch.f. widely used Gaussian random process is shown in Figure 1.2. It's plotted for the function

$$\theta_1(V_m) = \exp\left[j\, m_1 V_m - \left(\frac{\sigma^2 V_m^2}{2}\right)\right], \tag{1.12}$$

for which the coefficient m_1 =0. When σ=1, the value of ch.f. for V_m = 4 differs from $\theta_1(0)$ = 1 by a factor of 300. Therefore, ch.f. of normal random process rapidly decreases to zero. This behavior ch.f. is determined by its noise filtering property, which will be discussed below. In addition, the ch.f. has

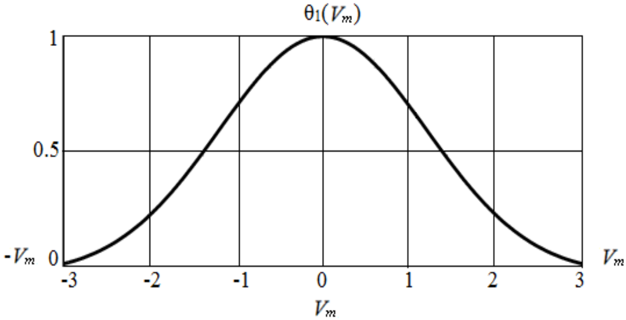


Figure 1.2. *Characteristic function of a Gaussian process (m_1=0, σ=1)*

other properties, for example, its maximum value is equal to one at V_m = 0 and the modulus $|\theta_1(0)|$ = 1. This implies the measurability and boundedness of the ch.f. if all values V_m [-∞,+∞].

When solving many problems, it is especially useful to have the property that the ch.f. additive mixture of signal and noise is equal to the product of the characteristic functions of individual terms. This property is easy to extend to the sum $\sum_{i=1}^{n} \xi_i(t)$ of independent random processes. Ch.f. of the amount will be

$$\theta_1(V_m) = \prod_{i=1}^{n} \theta_{1i}\,(V_m), \tag{1.13}$$

where $\theta_{1i}(V_m)$ is the ch.f. *i*-th random process. Here we can also talk about the probability density of the sum of independent random processes, which, as you know, is calculated by the convolution formula

$$W_1(z) = \int_{-\infty}^{+\infty} W_1(u) W_1(z-u) du = \\ = \int_{-\infty}^{+\infty} W_1(z-y) W_1(y) dy, \tag{1.14}$$

where *z(t)=u(t)+y(t)*. Then, using transformation (1.5), we can pass to the characteristic function of the additive mixture *z(t)*. However, these two options for solving the problem do not reject each other. In some cases, it turns out to be easier to find the probability density, in others, the characteristic function. For example, when measuring the probability density of only a signal from a mixture *z(t)*, of course, it is much easier to measure the ch.f. mixtures of *z(t)* and ch.f. interference *y(t)* when the signal *u(t)*=0. Then, it is necessary to find the ch.f. signal from the ratio of the characteristic functions, and to calculate its probability density using the Fourier transform

$$W_1(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \theta_1(V_m)\, e^{-jV_m x} dV_m\,, \qquad (1.15)$$

The solution of such a problem with the help of convolution (1.14) is much more complicated.

If we divide the characteristic function of the additive mixture by the ch.f. interference, we get the quotient. It contains expressions for the real and imaginary parts of the ch.f. separate signal. Expressions are equal

$$A_u(V_m) = \frac{A_z(V_m)A_y(V_m) + B_z(V_m)B_y(V_m)}{A_y^2(V_m) + B_y^2(V_m)}, \qquad (1.16)$$

$$B_u(V_m) = \frac{B_z(V_m)A_y(V_m) - A_z(V_m)B_y(V_m)}{A_y^2(V_m) + B_y^2(V_m)}, \qquad (1.17)$$

where $A_u(V_m)$, $A_y(V_m)$, $Az(V_m)$ is the real part of the ch.f. of signal, interference and additive mixture, respectively; $B_u(V_m)$, $B_y(V_m)$, $B_z(V_m)$ is the imaginary part of the ch.f. of signal, interference and additive mixture, respectively. When the noise has a symmetric distribution function and mathematical expectation equal to zero, then the calculation algorithms (1.16), (1.17) become simpler:

$$A_u(V_m) = \frac{A_z(V_m)}{A_y(V_m)}, \qquad (1.18)$$

$$B_u(V_m) = \frac{B_z(V_m)}{A_y(V_m)}. \qquad (1.19)$$

In accordance with (1.2), the ch.f. signal will be equal to

$$\theta_{1u}(V_m) = \frac{A_z(V_m)}{A_y(V_m)} + \frac{jB_z(V_m)}{A_y(V_m)}. \qquad (1.20)$$

If the additive mixture of signals is represented as $z(t) = K_{0u}(t) + U_0$, then the ch.f. sum looks like this:

$$\theta_{1z}(V_m) = \theta_{1u}(K_0 V_m)e^{jU_0 V_m}, \qquad (1.21)$$

where U_0 is deterministic signal, constant in time (for example, constant voltage); $\theta_{1z}(V_m)$ is ch.f. process z(t), and $\theta_{1u}(V_m)$ is ch.f. of signal $u(t)$. Where $U_0 = 0$, expression (1.21) is transformed into the equality

$$\theta_{1z}(V_m) = \theta_{1u}(K_0 V_m). \tag{1.22}$$

Consequently, amplification (attenuation) of the signal by a factor of K_0 leads only to a change in the real parameter of the ch.f. the same number of times. This property of the ch.f. is especially important when constructing instruments of a new class with the conditional name characterometers. When the sign of the real parameter of the ch.f. equality (1.22) takes the form

$$\theta_{1z}(-V_m)=\overline{\theta_{1u}(K_0 V_m)} \tag{1.23}$$

i.e. $\theta_{lz}(-V_m)$ and $\theta_{1u}(K_0 V_m)$ are complex conjugate functions. If we put $K_0 = 1$ in expression (1.23), then it is reduced to the known

$$\theta_{1z}(-V_m)= \overline{\theta_{1u}(V_m)}=A(V_m) - j\,B(V_m). \tag{1.24}$$

Let us onsider a complex signal

$$\xi(t) = u(t) + j\nu(t),$$

in which the terms are functions of a real variable. Then

$$\theta_{1\xi}(V_m) = m_1\{\exp(-V_m \nu(t))\}\theta_{1u}(V_m), \tag{1.24a}$$

where $\theta_{1\xi}(\cdot)$, $\theta_{1u}(\cdot)$ are the characteristic functions of the signals $\xi(t)$, $u(t)$, respectively. Thus, ch.f. of complex signal is equal to the product of the ch.f. of the real part of this signal and the expectation of the exponential function, whose exponent with the opposite sign is the imaginary part of the complex signal amplified by V_m times. If a random signal $v(t)$ is subject to the distribution law $W_1(x)$, then

$$m_1\{\exp(-V_m \nu(t))\} = -\int_{-\infty}^{+\infty} W_1(x)\, e^{-V_m x} dx. \tag{1.24б}$$

For example, for a law of the form $W_1(x)=1/(2\pi)$ we have

$$\theta_{1\xi}(V_m) = -\frac{\mathrm{sh}(\pi V_m)}{\pi V_m}\theta_{1u}(V_m). \tag{1.24в}$$

Multidimensional ch.f. of random process has the following form:

$$\theta_n(V_1, V_2, \dots, V_n, t_1, t_2, \dots, t_n) = m_1\left\{\exp\left[j\sum_{i=1}^{n} V_i \xi(t_i)\right]\right\}. \tag{1.25}$$

The quantization step ΔV is applicable to any ch.f. parameter. Taking into account the previously adopted notation, we can write

$$V_1 = V_{1m} = m\Delta V,\ V_2 = V_{2m} = m\Delta V, \dots,$$
$$V_n = V_{nm} = m\Delta V. \tag{1.26}$$

Moreover, it is not necessary at all in (1.26) to have a constant quantization step for all parameters of the ch.f. Each parameter (or group of parameters) ch.f. can be discretized by its step.

If the instantaneous values of the random process, taken at the moments of time $t_1, t_2, \dots, t_n$, do not depend on each other, then the expression (1.25) takes a different form:

$$\theta_n(V_1, V_2, \dots, V_n, t_1, t_2, \dots, t_n) = \prod_{i=1}^{n} \theta_{1i}(V_i), \tag{1.27}$$

where $\theta_{1i}(V_i)$ is ch.f. of *i*-th set of instantaneous values of a random process.

For stationary random processes, expression (1.25) is written in a simpler way:

$$\theta_n(V_1, V_2, \dots, V_n) = m_1 \left\{ \exp\left[j \sum_{i=1}^{n} V_i \xi(t_i) \right] \right\}, \tag{1.28}$$

since ch.f. does not depend on time or depends only on the differences of individual moments of time $t_2 - t_1, t_3 - t_1, \dots, t_n - t_1 = n\tau$, i.e.

$$\theta_n(V_1, V_2, \dots, V_n, \tau, 2\tau, \dots, (n-1)\tau) =$$
$$= m_1 \left\{ \exp\left[j \sum_{i=1}^{n} V_i \xi(t + i\tau) \right] \right\}. \tag{1.29}$$

It is quite clear that the ch.f. of lower dimension is quite simply obtained from the *n*-dimensional characteristic function, namely:

$$\theta_k(V_1, V_2, \dots, V_k) = \theta_n(V_1, V_2, \dots, V_n, 0, 0, \dots, 0). \tag{1.30}$$

Calculation of *n*-dimensional ch.f. random process is a difficult task. Therefore, the publication contains few examples of multidimensional ch.f. Nevertheless, the expression for the n-dimensional ch.f. normal random process:

$$\theta_n(V_1, V_2, \dots, V_n) =$$
$$= \exp\left[j \sum_{i=1}^{n} V_i m_{1i} - \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} V_i V_k \sigma_i \sigma_k R_{ik}(\tau) \right], \tag{1.31}$$

where m_1, σ^2 are the mathematical expectation and variance of the random process; $R_{ik}(\tau)$ is the normalized correlation function, with $R_{ik}(\tau)=R_{ki}(\tau)$, $R_{ii}(0) = 1$, $R_{kk}(0)=1$. Examples of other multidimensional ch.f. can be found in the books [2,5].

1.2. Estimates of the characteristic function of instantaneous values of a random process

Formula (1.1) contains the mathematical operation $m_1\{\cdot\}$ is the statistical mean (or the expectation operator), which involves averaging an infinitely large number of values of the function $\exp[jV_m\xi(t)]$, depending on the instantaneous values of the random process $\xi(t)$.

It is quite clear that in the instrumental analysis of ch.f. a finite number of instantaneous values of the random process or its parameters (envelope, instantaneous phase, frequency) will be used. The result of calculating the value of ch.f. according to a limited set of sample data[1] of a random process is called the estimate of the function. The estimate of the ch.f. will be denoted by the previously adopted symbol, and to distinguish it, mark with an asterisk:

$$\theta_1^*(V_m) = L\{\exp[jV_m\xi_i(t_n)]_1^k\}, \tag{1.32}$$

$$\theta_1^*(V_m) = L\{\exp[jV_m\xi(t_n)]_1^k\}, \tag{1.33}$$

where L is the transformation operator for an array of sample data, a set of sample values, or time-limited implementations of a random process; k is the number of implementations or sequences used[2].

The transformation operator L can be different. In [6], three operators are considered, namely: an ideal integrator with normalization with respect to T

$$L_T = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\,, \tag{1.34}$$

ideal adder normalized by N

$$L_N = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\ , \tag{1.35}$$

ideal adder-integrator with normalization on N and T

$$L_{NT} = \lim_{N,T\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\int_0^T dt\,, \tag{1.36}$$

where T is the averaging time (the duration of the implementation of the random

[1]The instantaneous value of the implementation of a random process is called the sample value and is denoted by the symbol $\xi_i(t_n)$ - the value of the i-th implementation at the time t_n.

[2]The set of instantaneous values corresponding to the values of the i-th realizations at the same time t_n is called the n^{th} sequence of the process $\xi(t)$ and denoted by $\xi(t_n)$.

process); N is the amount of sample data or the set of sample values. Taking into account (1.34) - (1.36), we can write:

$$\theta_1^*(V_m) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\exp\left[jV_m\xi_i(t_n)\right], \tag{1.37}$$

$$\theta_1^*(V_m) = \lim_{T\to\infty}\frac{1}{T}\int_0^T\exp\left[jV_m\xi_k(t)\right]dt, \tag{1.38}$$

$$\theta_1^*(V_m) = \lim_{N,T\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\int_0^T\exp\left[jV_m\xi_i(t)\right]dt. \tag{1.39}$$

By definition [6], estimates (1.37) - (l.39) are called k - current, t - current and average, respectively. The application of these estimates is largely determined by the type of random process. Using the classification [4] of random processes, we can specify the following:

- for a stationary ergodic random process, all estimates of the ch.f. are equal to each other;
- for a stationary non-ergodic random process t - current and average estimates of ch.f. are equal to each other;
- for a non-stationary ergodic random process k - current and average estimates of the ch.f. are equal to each other;
- for a non-stationary non-ergodic random process, all estimates of the ch.f. are not equal to each other. In this case, we recommend using average ch.f. estimates, since they converge better than others to a probabilistic characteristic (characteristic function).

Passing to the estimates of the real and imaginary parts of the ch.f., taking into account (1.37) – (1.39), we write

$$A^*(V_m) = \lim_{T\to\infty}\frac{1}{T}\int_0^T\cos[V_m\xi_k(t)]dt, \tag{1.40}$$

$$B^*(V_m) = \lim_{T\to\infty}\frac{1}{T}\int_0^T\sin[V_m\xi_k(t)]dt, \tag{1.41}$$

$$A^*(V_m) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\cos[V_m\xi_i(t_n)], \tag{1.42}$$

$$B^*(V_m) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \sin[V_m \xi_i(t_n)]\,, \tag{1.43}$$

$$A^*(V_m) = \lim_{N,T\to\infty} \frac{1}{NT}\sum_{i=1}^{N}\int_0^T \cos\,[V_m \xi_i(t)]dt\,, \tag{1.44}$$

$$B^*(V_m) = \lim_{N,T\to\infty} \frac{1}{NT}\sum_{i=1}^{N}\int_0^T \sin[V_m \xi_i(t)]dt\,. \tag{1.45}$$

Our further analysis is connected with the *k*-current estimate of the ch.f. To simplify the notation, we omit the symbols *k*, n in formulas (1.37), (1.40), (1.41), and we will only have them in mind. If in estimates (1.37), (1.40), (1.41) the integration operation is performed by a non-ideal integrator, and a real device with an impulse response *q(t)*, then these expressions take the form

$$\theta_1^*(V_m) = \int_0^T q(T-t)\exp\left[jV_m\,\xi(t)\right]dt, \tag{1.46}$$

$$A^*(V_m) = \int_0^T q(T-t)\cos[V_m\xi(t)]dt, \tag{1.47}$$

$$B^*(V_m) = \int_0^T q(T-t)\sin[V_m\xi(t)]dt. \tag{1.48}$$

Let us extend the notion of an estimate to an *n*-dimensional ch.f. Similarly to estimates (1.37)–(1.39), let us write

$$\theta_n^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T \exp\,[j(V_1+V_2+\dots+V_n)\xi_k(t)]dt\,, \tag{1.49}$$

$$\theta_n^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} \exp\,[j(V_1+V_2+\dots+V_n)\xi_i(t_n)]\,, \tag{1.50}$$

$$\theta_n^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N,T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \int_0^T \exp\,[j(V_1 + V_2 + \dots + V_n)\xi_i(t)]dt\,. \tag{1.51}$$

Separately for the real and imaginary parts of the ch.f. we have

$$A^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T \cos[(V_1 + V_2 + \dots + V_n)\xi_k(t)]dt\,, \tag{1.52}$$

$$B^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T \sin[(V_1 + V_2 + \dots + V_n)\xi_k(t)]dt\,, \tag{1.53}$$

$$A^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \cos[(V_1 + V_2 + \dots + V_n)\xi_i(t_n)]\,, \tag{1.54}$$

$$B^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \sin[(V_1 + V_2 + \dots + V_n)\xi_i(t_n)]\,, \tag{1.55}$$

$$A^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N,T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \int_0^T \cos\,[(V_1 + V_2 + \dots + V_n)\xi_i(t)]dt\,, \tag{1.56}$$

$$B^*(V_1, V_2, \dots, V_n) =$$

$$= \lim_{N,T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \int_0^T \sin[(V_1 + V_2 + \dots + V_n)\xi_i(t)]dt\,. \tag{1.57}$$

For a *k*-current estimate of a multidimensional ch.f. taking into account (1.46) - (1.48) we can write

$$\theta_n^*(V_1, V_2, \dots, V_n) =$$

$$= \int_0^T q(T-t)\exp[j(V_1 + V_2 + \dots + V_n)\xi(t)]\,dt, \qquad (1.58)$$

$$A^*(V_1, V_2, \dots, V_n) =$$

$$= \int_0^T q(T-t)\cos[(V_1 + V_2 + \dots + V_n)\xi(t)]dt, \qquad (1.59)$$

$$B^*(V_1, V_2, \dots, V_n) =$$

$$= \int_0^T q(T-t)\sin[(V_1 + V_2 + \dots + V_n)\xi(t)]dt. \qquad (1.60)$$

In the above formulas, the model of a random process can be any, and the model of the impulse response of a physically realized integrator is presented in Table 1.1.

In the theory of estimating the probabilistic characteristics of a random process, much attention is paid to the fundamental nature of estimates. Fundamentality is characterized by properties of estimates. They must be wealthy, efficient and unbiased. The properties of estimates (1.40 - 1.45, 1.47, 1.48) are studied in detail and described in the book [2]. It is also shown there that the estimates for the real and imaginary parts of the ch.f. are consistent, efficient, and asymptotically unbiased, i.e. they are fundamental.

Table 1.1.

Classification of integrators

Integrator type	Characteristic q(T-t)
RC -integrator	$\beta e^{\beta(t-T/2)}/2sh(\beta T/2)$
RC -integrator	$(l-\beta T+\beta t)e^{\beta t}/T$
RC -integrator	$2(T-t-\beta e^{\beta(t-T)})/(2e^{-\beta T}+T^2-2)$
Measuring instrument with critical damping	$\beta^2(T-t)e^{\beta T}/(e^{\beta T}-\beta T-1)$

Turning to estimates of the form (1.36 - 1.39, 1.46), we can say that they are also fundamental. Their properties will be the same as those of the estimates (1.40,1.41,1.47,1.48) of the real and imaginary parts of the ch.f. Here it is appropriate to say the following. Estimates (1.40), (1.41), (1.47), (1.48) are assigned the role of some approximators, with the help of which estimates of the probability density, correlation function, initial and central moment functions are constructed.

Since the initial estimates (approximators) of the probabilistic characteristics of a random process are consistent, other estimates of the probabilistic characteristics constructed from them by classical methods will also be consistent, which is consistent with the conclusion in [2].

In the instrumental analysis of random processes, it can be difficult to obtain an estimate of a probabilistic characteristic that satisfies all of the above properties at once. For example, it may turn out that even if an effective estimate exists, then the algorithms for calculating it are too complicated and one has to be content with a simpler estimate, the variance of which is somewhat larger. The same can be said about biased estimates, as slightly biased estimates are sometimes used. The final choice of the estimate, as a rule, is dictated by the results of its analysis in terms of simplicity, ease of use in equipment or mathematical statistics, and reliability of properties. Estimates of the probability distribution function, probability density, correlation function, initial and central moment functions of the kth order are given in the book [2].

1.3. Estimation of the characteristic function of a discrete quantity

Let us consider a special case when the process $\xi(t)$ is represented only by instantaneous values $\xi_i(tn)$. To simplify the formulas, we introduce the notation $\xi_i(t_n) = \xi_i$. This is a discrete random variable for which the probability density has the form [4]

$$W_1(\xi) = \sum_{i=1}^{N} p_i \delta(\xi - \xi_i), \tag{1.61}$$

where p_i is the probability of occurrence of the i-th value of a discrete random variable; N is the total number of discrete values of a random variable; δ (·) is the delta function [4]. In view of (1.61), the ch.f. discrete random variable

$$\theta_1(V_m) = \sum_{i=1}^{N} p_i \exp(jV_m \xi_i)\,. \tag{1.62}$$

If a discrete random variable $\xi_i = U_0 =$ const with probability p=1, then the ch.f. constant value is calculated by the formula

$$\theta_1(V_m) = \exp(jV_m U_0). \tag{1.63}$$

Known distribution laws for a discrete random variable are tabulated in the book [5]. The characteristic functions obtained taking into account the known distribution laws of a discrete quantity are also given there. For example, a discrete random variable distributed according to the Poisson law has a ch.f. kind

$$\theta_1(V_m) = \exp\left[\lambda\left(e^{jV_m} - 1\right)\right], \tag{1.64}$$

where λ – Poisson distribution parameter. This function is widely used when observing the flows of elementary particles, such as electrons.

Formulas (1.7) - (1.11) can be used to calculate the distribution moments of a discrete random variable. Substituting the ch.f. (1.64) into formula (1.7), we obtain

$$m_1\{\xi_i\} = \lambda, \quad m_2\{\xi_i\} = \lambda^2 + \lambda, \quad M_2\{\xi_i\} = \lambda^2 \,. \tag{1.65}$$

They coincide with the results contained in [4]. Thus, the above material regarding ch.f. random process can be extended to a discrete random variable, the ch.f. which is defined by formula (1.32).

Let us consider a complex number $\xi_i = ui + jvi$, which, in accordance with the notation adopted earlier, can be called a complex discrete quantity. Similarly to (1.24a), we obtain

$$\Theta_{1\xi}(V_m) = m_1\{\exp(-V_m \nu_i)\}\Theta_{1u}(V_m) \,. \tag{1.66}$$

Let's denote $y_i = \exp(-V_m \nu_i)$. Then we have

$$m_1\{y_i\} = \frac{1}{N}\sum_{i=1}^{N} p_i y_i, \tag{1.67}$$

where p_i is the probability with which the random variable takes on the value y_i. If the probability density of this quantity is

$$W_1(y) = \sum_{i=1}^{N} p_i \delta(y - y_i), \tag{1.68}$$

and the probability density of the random variable v is equal to

$$W_1(\nu) = \sum_{i=1}^{N} \ddot{p}_1 \delta(\nu - \nu_i), \tag{1.69}$$

then we get the equation

$$p_i = \frac{1}{V_m}\ddot{p}_1 \exp(V_m \nu_i). \tag{1.70}$$

Taking into account (1.67), (1.70), we have the ch.f. discrete random variable

$$\Theta_{1\xi}(V_m) = \frac{1}{NV_m}\sum_{i=1}^{N} \ddot{p}\Theta_{1u}(V_m). \tag{1.71}$$

When the probability is $\ddot{p} = 1/N$, then

$$m_1\{\exp(-V_m \nu_i)\} = 1/V_m, \tag{1.72}$$

where N is the number of equiprobable values of the discrete random variable v. For this case, expression (1.71) takes on the form

$$\theta_{1\xi}(V_m) = \frac{1}{V_m}\theta_{1u}(V_m). \qquad (1.73)$$

New formulas of the form (1.73) follow from consideration of other laws of distribution of a discrete random variable.

1.4. New property of the characteristic function

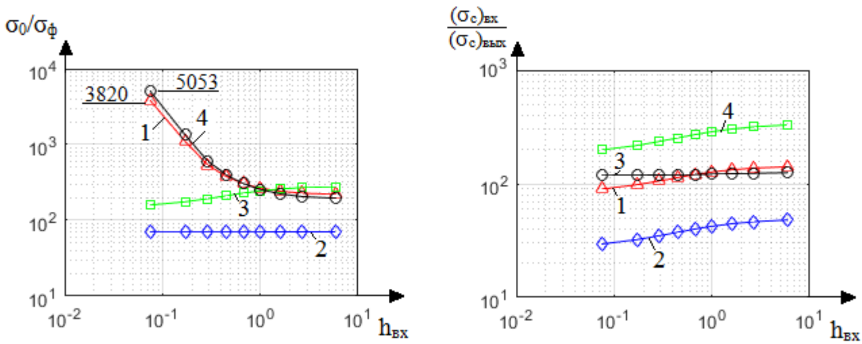
In the above analysis of the characteristic function, special attention was paid to its properties known from the publication, such as, for example, boundedness, measurability, and others. We have discovered a new property of ch.f., which concerns filtering in the space of noise probabilities with the help of this function.

Expressions (1.16 - 1.19) form the basis of the filtering capacity of the characteristic function. **Filtering capacity** of ch.f. has an applied value [3,7] and the following physical meaning.

The probability density of a random process and its ch.f. are connected by a pair of Fourier transforms (1.5), (1.15). It turns out that the ch.f. is the spectral density of the probability density (or, in short, the spectral density of the probabilities) of a random process in the domain of probabilities, if we use the terminology of the Fourier transform of signals, in which the domain is called the frequency domain. Ch.f. carries information about the probabilities of occurrence of instantaneous values of a random process depending on the parameter V_m, which we previously proposed to call the multiplicity of values of a random process. This multiplicity can be written as integer and fractional real numbers. For integer multiplicities, $V_m = \pm1, \pm2, \ldots, \pm\infty$, and for fractional multiplicities, V_m takes any other values on the real axis from $-\infty$ to $+\infty$. This is done in the same way as in the frequency domain, when imaging a line spectrum, the spectral lines are located at points with abscissas $\pm\omega, \pm2\omega, \pm3\omega, \ldots, \pm n\omega$, where ω is the circular frequency of the signal. By analogy with the physical spectrum when using the ch.f. in practice, the multiplicity V_m is taken to be integer and positive. At V_m=0 ch.f. $\Theta_1(0)=1$ is the total probability. This total probability is distributed between the probabilities of presence in the signal of instantaneous values with a multiplicity one (V_m=1), with a multiplicity two (V_m=2), with a multiplicity three (V_m=3), etc. For example, for "white" noise with dispersion σ^2 =1 and ch.f. of the form (1.12) at m_1 = 0 we have: p_1 =0,6065 at V_m =1; p_2 =0,1353 at V_m =2; p_3 =0,0111 at V_m =3 etc. to infinity, and for V_m =∞ the probability p_∞ =0. When filtering the additive mixture using expressions (1.16 - 1.19) with V_m =1, all instantaneous noise values that are present with a probability p_1 =0.6065 are “cut out” from it. In this case, after filtering, instantaneous noise values remain in the additive mixture with a total probability of 0.3935, i.e. with a probability less than one. At the filter output, the additive mixture is different, it will be partially “cleaned” of noise. And, as a result of this, the signal-to-noise ratio at the filter output increases.

You can continue filtering further the "purified" additive mixture, "cutting out" from it at $V_m = 2$ the instantaneous noise values that are present with a probability $p_2 = 0.1353$. Then, at the output of the second filter, the signal-to-noise ratio increases even more, and the additive mixture becomes “cleaned” twice. And so it should be continued further, changing only the value of the multiplicity $V_m = m\Delta V$. When $\Delta V=1$, it is appropriate to use the well-known term "filter section" and write the definition of a new device in the form *m* - link virtual filter.

Simulation of virtual filters using the characteristic function with $V_m = 1$ showed good results. The works [3,7] present quantitative and graphical data obtained using equation (1.19). As an example, Figure 1.3 shows graphs of noise and signal suppression when filtering an additive mixture, where σ_0, σ_f σf are the SNR at the input and output of the filter, respectively; σ_s is the SNR of the signal; $h_{вх}$ is the signal-to-noise ratio at the filter input. The designation *N* is taken from expressions (1.42,1.43). An analysis of the curves in Figure 1.3 shows that the filter suppresses signal and noise differently. "White" noise with the help of ch.f. is suppressed by a factor of 5053, while the quasi-deterministic signal is attenuated by only a factor of 120. Thus, the signal-to-noise ratio at the output of a single-link virtual filter increased by an average of 30 times.



a) b)

Figure 1.3. *Filter suppression of noise (a) and signal (b):*
1, 4 – $N = 5$; 2 – $N = 10$; 3 – $N = 50$

2. QUASI-DETERMINISTIC SIGNALS

To implement statistical modulation, quasi-deterministic signals are required, which, by definition [4 p. 171] ".... are described by time functions of a given type, containing one or more random parameters ζ_1, ζ_2, ζ_3..., that do not depend on time." From this class of signals, the model and probabilistic characteristics of a quasi-deterministic signal with the arcsine distribution law are quite fully presented in the publication. Information about other quasi-deterministic signals is presented only fragmentarily.

2.1. Signal model with arcsine distribution law

Let us consider a signal with a mathematical model of the form

$$u(t) = a\sin(\omega_0 t + \eta), \tag{2.1}$$

where a is the constant amplitude of the signal; ω_0 - constant circular frequency of the signal; η - a random variable (initial phase angle) with a uniform distribution law within - π . . π; $u(t)$ instantaneous values of the signal, obeying the distribution law of the arcsine. In the publication, this signal is called quasi-deterministic.

We begin the description of probabilistic characteristics with the probability density of instantaneous values of the signal, which, by definition [4], is equal to

$$W_1(x) = \frac{1}{\pi\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{2}\pi\sigma\sqrt{1-\left(\frac{x}{a}\right)^2}}, \tag{2.2}$$

where $\sigma^2 = a^2/2$ dispersion (average power) of the signal. Here and below, the number 1 denotes the one-dimensionality of the function. The graph of function (2.2) is shown in Figure 2.1.

Let's take a look at the shape of the graph. It looks like a horseshoe and is centered around zero because the mathematical expectation of the signal is zero. At the edges, the value of the function tends to infinity when the value of the argument is equal to the amplitude of the signal. This results in the probability of occurrence of the signal amplitude as if equal to one, since this is like depicting a probability density of a constant value using a delta function $\delta(\cdot)$. However, this is possible, and this is explained only by the fact that the signal model (2.1) is some mathematical abstraction. The physical process has amplitude fluctuations [8], which are random. And as

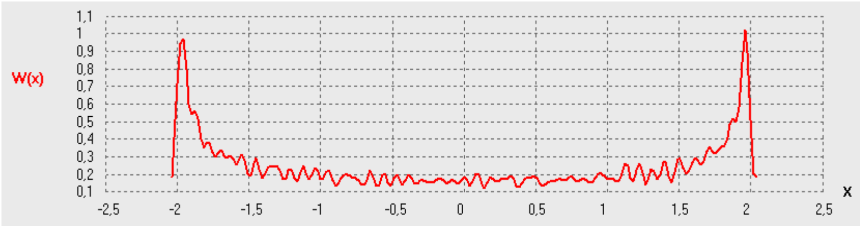


Figure 2.1. *Signal probability density*

a result of this, the specified equality $a=const$ is violated. The arcsine law was established in 1939 by the mathematician P. Levy for random walks of a point on a straight line and later adapted for signal (2.1).

The characteristic function of the signal in accordance with (1.5) is equal to

$$\Theta_1(V_m)=\int_{-\infty}^{\infty} W_1(x)\exp(jV_m x)dx = J_0(V_m a), \tag{2.3}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind (see Fig. 2.2). For a random variable η with probability density $W_1(\eta)=(1/2\pi)$ ch.f. will be [4]

$$\Theta_1(V_m)=\frac{\sin(V_m\pi)}{V_m\pi}\ . \tag{2.4}$$

For a constant signal amplitude ch.f. was defined earlier with the help of expression (1.63), which we repeat with our designations

$$\Theta_1(V_m)=\exp(jV_m a). \tag{2.5}$$

The signal correlation function (2.1) will be

$$k_u(\tau)=\int_{-\pi}^{\pi} u(t)u(t+\tau)W(\eta)d\eta = \frac{1}{2}a^2\cos\omega_0\tau = \sigma^2\cos\omega_0\tau, \tag{2.6}$$

where $W(\eta)$ is the probability density of the random phase η; τ is the shift in time.

The dependence of the average signal power on time turns out to be harmonic, it is shown in Figure 2.3.

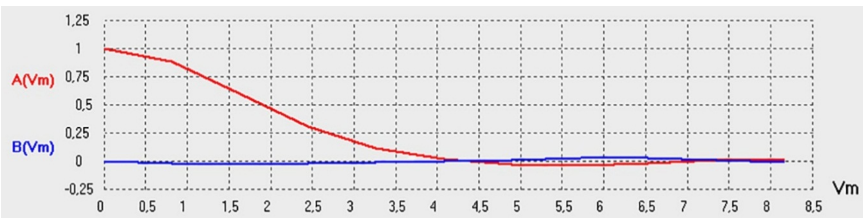


Figure 2.2. *Ch.f. of signal (separate real and imaginary part)*

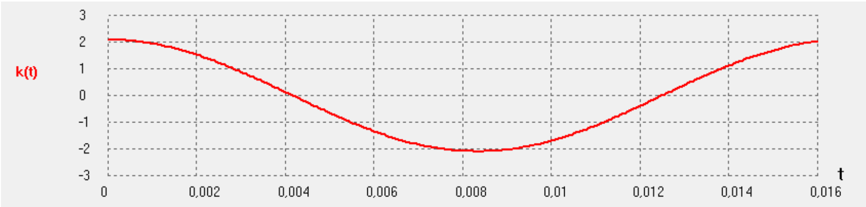


Figure 2.3. *Correlation function of a signal with a frequency of 30 Hz*

Let us proceed to the analysis of the power spectral density (energy spectrum) of the signal (2.1). Let's write down its energy spectrum

$$G_u(\omega)=\int_{-\infty}^{\infty}k_u(\tau)\exp(-j\omega\tau)d\tau=\frac{\pi}{2}\sigma^2[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \ . \qquad (2.7)$$

The spectrum (2.7) turned out to be lined. It contains a spectral component r $\delta(\omega-\omega_0)$ anging from - ∞ to 0 and a spectral component $\delta(\omega-\omega_0)$ ranging from 0 to ∞, where $\delta(\cdot)$ – delta is a function. In the transition to the physical spectrum, i.e. to the spectrum in the region of positive frequencies, we get

$$G_u(\omega)=\pi\sigma^2\delta(\omega+\omega_0) \ . \qquad (2.8)$$

The energy spectrum as a function of frequency is shown in Figure 2.4.

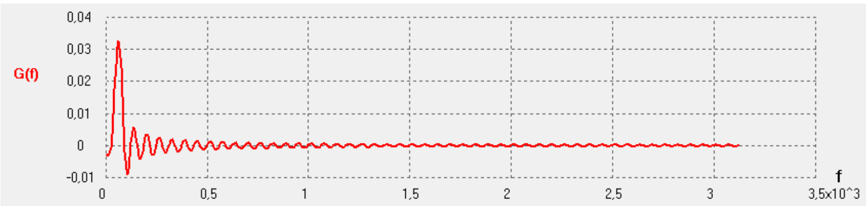


Figure 2.4. *Signal power spectral density at 30 Hz*

To clarify, Figures 2.1–2.4 show estimates of the probabilistic characteristics of the signal (2.1) measured using the virtual instrument "Characteriometer" [3, 9]. Signal (2.1) was obtained from the output of the G3-54 generator.

2.2. Signal model with the Veshkurtsev distribution law

Let's repeat the mathematical model of the signal (2.1)

$$u(t)= a \sin (\omega_{0t} +\eta), \qquad (2.9)$$

where *a, η* - random variables (amplitude and initial angle of phase shift, respectively); ω_0 - constant circular frequency of the signal; *u(t)* - instantaneous signal values distributed according to the Veshkurtsev law [10]. There is no description of such a signal in the publication.

Let the signal amplitude (2.9) be distributed according to the Gauss law (normal law)

$$W_1(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} , \tag{2.10}$$

and the random phase - according to a uniform law within – π . .π, where σ^2 is the amplitude dispersion. Then the instantaneous values of the signal obey the Veshkurtsev distribution

$$W_1(x) = \frac{1}{\sqrt{2\pi^3}\sigma} \exp\left(-\frac{x^2}{4\sigma^2}\right) K_0\left(\frac{x^2}{4\sigma^2}\right), \tag{2.11}$$

where K_0 (·) is the cylindrical function of the imaginary argument (the Macdonald function) [11]. The MacDonald function at $x = 0$ asymptotically tends to infinity on both sides of the y-axis (Fig. 2.5). In this way, Veshkurtsev's law resembles the arcsine law (Fig. 2.1), where the value of the probability density tends to infinity at the edges when the argument value is equal to the signal amplitude. It turns out that Veshkurtsev's law is a kind of copy of the transformed arcsine law, therefore, it is also some kind of mathematical abstraction. A physical process with such a law does not exist, and only digital technology will make it possible to put it into practice in the form of a random process sensor.

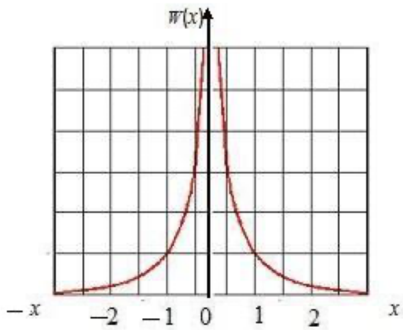


Figure 2.5. *Signal Probability Density*

Since this law was obtained for the first time, we will agree to call it the **Veshkurtsev law** in the future by the name of the author, who was the first to write it down analytically and apply it in practice in solving new problems [10,12,13]. Naturally, all the properties of the statistical law prescribed in the theory of probability have been verified by the author and they are fulfilled. Using the Fourier transform of this law, we obtain the ch.f. signal

$$\Theta_1(V_m) = \int_{-\infty}^{\infty} W_1(x)\exp(jV_m x)dx = I_0\left(\frac{V_m^2\sigma^2}{4}\right)\exp\left(-\frac{V_m^2\sigma^2}{4}\right), \tag{2.12}$$

where $I_0(\cdot)$ is the Bessel function of the imaginary zero-order argument. A similar transformation of the distribution law (2.10) gives the ch.f.

$$\Theta_1(V_m) = \exp\left(-\frac{1}{2}V_m^2\sigma^2\right) \tag{2.13}$$

of signal amplitude with a Gaussian distribution law. Ch.f. for the random phase of the signal will be

$$\Theta_1(V_m) = \frac{\sin(V_m\pi)}{V_m\pi} \, . \tag{2.14}$$

Quasi-deterministic signal (2.9) is centered (its mathematical expectation is zero), it has dispersion (average power)

$$\sigma_c^2 = \frac{\sigma^2}{2} {}_2F_2\left(\frac{3}{2},\frac{3}{2};\frac{1}{2},2;0\right) = \frac{9\sigma^2}{8}, \tag{2.15}$$

where ${}_2F_2(\cdot)$ - generalized hypergeometric series [11]; σ^2 - signal amplitude dispersion.

Concluding the analysis of the probabilistic characteristics of the quasi-deterministic signal (2.9), we clarify that its instantaneous values are distributed according to the Veshkurtsev law, the amplitude is distributed according to the Gauss law, and the phase is distributed according to the uniform law.

The signal correlation function (2.9) has the form

$$k_u(\tau) = \int_{-\infty}^{\infty}\int_{-\pi}^{\pi} u(t)\times u(t+\tau)W_1(x)W(\eta)d\eta dx = \frac{\sigma^2}{2}\cos\omega_0\tau. \tag{2.16}$$

The energy spectrum of the signal (2.9) coincides with the spectrum (2.7)

$$G_u(\omega) = \int_{-\infty}^{\infty} k_u(\tau)\exp(-j\omega\tau)d\tau = \frac{\pi}{2}\sigma^2[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \, . \tag{2.17}$$

In the transition to the physical spectrum, i.e. to the spectrum in the region of positive frequencies, we obtain

$$G_u(\omega) = \pi\sigma^2\delta(\omega+\omega_0) \, . \tag{2.18}$$

The physical spectrum of the signal with the Veshkurtsev distribution law contains only one spectral component located on the frequency axis at the point with the abscissa ω_0, when $\omega = 0$, and coincides with the origin.

2.3. Signal model with cosine distribution law

Let's repeat the mathematical model of the signal (2.1)

$$u(t) = a\sin(\omega_0 t + \eta), \tag{2.19}$$

where α, η - random variables (amplitude and phase shift angle, respectively), each with its own distribution law; ω_0 - constant circular frequency; $u(t)$ - instantaneous signal values obeying the distribution law

$$W_1(x) = B(\cos x)^{\upsilon-1} \quad \text{at} \quad B = \frac{\Gamma(\upsilon)}{2^{\upsilon-1}\Gamma^2\left(\frac{\upsilon}{2}\right)}, \text{ where } \upsilon > 0. \tag{2.20}$$

Here and below, the number 1 denotes a one-dimensional function. The statistical law (2.20) is given in the book [5 p. 46] without a title and additional explanations, there is no information about its use in the literature. Apparently, we are the first to pay attention to this statistical law. For further actions, we take the value in formula $\upsilon = 2$ (2.20), then $B = 1/2$, and expression (2.20) takes the form

$$W_1(x) = (1/2)\cos x \tag{2.21}$$

at $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. The statistical law (2.21) has all the properties prescribed in the probability theory. We will call it the law of cosine in the future. There are no quantitative parameters in the mathematical description of the law of **cosine**. It should be noted that this law is centered, the mathematical expectation is zero, and the dıspersion is

$$\sigma_c^2 = \frac{\pi^2}{4} - 2. \tag{2.22}$$

It is always constant and depends only on the bounds of the values of the variable x. By this, this law is inferior to the Gauss law (normal law), in which the dispersion and mean square deviation (MSD) are included in the mathematical description of the law.

If the instantaneous values of the quasi-deterministic signal (2.19) are distributed according to the cosine law, then the signal amplitude will be distributed according to the law [14]

$$W_1(y) = \frac{\pi \times \Gamma(0,95)}{2} y J_0\left(\sqrt{2}y\right) \quad \text{at} \quad 0 \le y \le \frac{\pi}{2}, \tag{2.23}$$

where $J_0(\cdot)$ - Bessel function of the zero order of the first kind; Γ (·) – gamma – function. Since this law was obtained by us for the first time, we will call it the **Bessel law** in the future by analogy with the function of the same name included in it. Using the Bessel law, we determine the initial moments of the distribution of the signal amplitude (2.19), while obtaining the initial moment of the first order (expectation) [14]

$$m_1\{a\} = \frac{\pi^2\Gamma(0,95)}{8}\left[J_0\left(\frac{\sqrt{2}\pi}{2}\right)S_{1,-1}\left(\frac{\sqrt{2}\pi}{2}\right) + J_1\left(\frac{\sqrt{2}\pi}{2}\right)S_{2,0}\left(\frac{\sqrt{2}\pi}{2}\right)\right] - \frac{\pi\Gamma(0,95)}{4\sqrt{2}} \tag{2.24}$$

and the initial moment of the second order [14]

$$m_2\{a\} = \frac{\pi^2\Gamma(0,95)}{8\sqrt{2}}\left[2J_0\left(\frac{\sqrt{2}\pi}{2}\right)S_{2,-1}\left(\frac{\sqrt{2}\pi}{2}\right) + J_1\left(\frac{\sqrt{2}\pi}{2}\right)S_{3,0}\left(\frac{\sqrt{2}\pi}{2}\right)\right], \tag{2.25}$$

where $S_{1,-1}(.), S_{2,-1}(.), S_{2,0}(.), S_{3,0}(.)$ - Lommel function [11]; $J_k(\cdot)$ is the Bessel function of the k^{th} order of the first kind [11].

Turning to the random phase of the signal (2.19), we say that as a result of mathematical calculations, we have obtained a uniform law according to which the phase is distributed within $-\frac{\pi}{2} \le \eta \le \frac{\pi}{2}$.

The characteristic function (ch.f.) of a centered quasi-deterministic signal (2.19) is [14]

$$\Theta_1(V_m) = \frac{\cos\left(V_m \frac{\pi}{2}\right)}{\left(1 - V_m^2\right)}. \tag{2.26}$$

The work [2] describes the properties of the ch.f., which the function (2.26) satisfies. In particular, one property of the ch.f. concerns the signal distribution law, from which it follows that the ch.f. for a signal with a non-centered distribution law $W_1(x \pm e_0)$ is equal to the ch.f. obtained for a signal with a centered distribution law $W_1(x)$, multiplied by the exponent $\exp(\mp jV_m e_0)$, where e_0 – the expectation of the signal. Let us use this property and write the function (2.26) for a non-centered quasi-deterministic signal (c.q.s.), i.e. a signal that has an expectation. As a result, we will have

$$\Theta_1(V_m) = \frac{\cos\left(V_m \frac{\pi}{2}\right)}{\left(1 - V_m^2\right)} \exp(jV_m e_0) = A(V_m) + jB(V_m), \tag{2.27}$$

where $A(V_m), B(V_m)$ are the real and imaginary parts of the ch.f. respectively. In contrast to (2.27), the ch.f. (2.26) has only a real part.

Passing to the ch.f. random amplitude of the centered signal (2.19), we have [14]

$$\Theta_1(V_m) = \frac{\Gamma(0{,}95)\pi^3}{24}\left[A(V_m) + jB(V_m)\right], \tag{2.28}$$

$$A(V_m) = J_1\left(\frac{\sqrt{2}\pi}{2}\right)\cos\left(\frac{V_m\pi}{2}\right) + J_2\left(\frac{\sqrt{2}\pi}{2}\right)\sin\left(\frac{V_m\pi}{2}\right);$$

$$B(V_m) = J_1\left(\frac{\sqrt{2}\pi}{2}\right)\sin\left(\frac{V_m\pi}{2}\right) - J_2\left(\frac{\sqrt{2}\pi}{2}\right)\cos\left(\frac{V_m\pi}{2}\right);$$

where $J_1(\cdot)$ - Bessel function of the first order of the first kind; $J_2(\cdot)$ - Bessel function of the second order of the first kind [11]. Expression (2.28) is a particular solution; it is valid for the value $V_m = \sqrt{2}$, while the general solution for the ch.f. signal amplitude (2.19) is still in the search stage. Since the expectation of the signal amplitude (2.19) is not equal to zero, the ch.f. (2.28) is a complex function.

For the random phase of the signal (2.19), the ch.f. known [4 p. 162] and is equal to

$$\Theta_1(V_m)=\frac{2\sin\left(\frac{\pi}{2}V_m\right)}{\pi V_m}, \quad (2.29)$$

it is a real function, since the phase distribution law is centered.

Concluding the analysis of the probabilistic characteristics of the quasi-deterministic signal (2.19), we clarify that its instantaneous values are distributed according to the cosine law, the amplitude - according to the Bessel law, and the phase - according to the uniform law.

The signal correlation function (2.19) will be

$$k_u(\tau)=\int_0^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u(t)u(t+\tau)W(y)W(\eta)dyd\eta=\frac{1}{2}m_2\{a\}\cos\omega_0\tau, \quad (2.30)$$

where $W(y)$ - amplitude probability density (2.23); $W(\eta)$ - probability density of the random phase η; $m_2\{a\}$ is the initial moment of the second order (2.25). Let us proceed to the analysis of the power spectral density (energy spectrum) of signals (2.19). Let's write the energy spectrum of the signal

$$G_u(\omega)=\int_{-\infty}^{\infty}k_u(\tau)\exp(-j\omega\tau)d\tau=\frac{\pi}{2}m_2\{a\}\times[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]. \quad (2.31)$$

The spectrum (2.31) turned out to be lined. It contains a spectral component $\delta(\omega-\omega_0)$ ranging from - ∞ to 0 and a spectral component $\delta(\omega-\omega_0)$ ranging from 0 to ∞, where $\delta(\cdot)-$ where delta is a function. In the transition to the physical spectrum, i.e. to the spectrum in the region of positive frequencies, we obtain

$$G_u(\omega)=\pi\times m_2\{a\}\delta(\omega+\omega_0). \quad (2.32)$$

Like the signal (2.9), the physical spectrum of a signal with a distribution according to the cosine law contains only one spectral component located on the frequency axis at the point with the abscissa ω_0, when $\omega = 0$ and coincides with the origin of coordinates.

2.4. Signal model with the Tikhonov distribution law

Let's repeat the mathematical model of the signal (2.1)

$$u(t)=a\sin(\omega_0 t+\eta), \quad (2.33)$$

where *α, η* - random variables (amplitude and phase shift angle, respectively), each with its own distribution law; ω_0 - constant circular frequency; $u(t)$ - instantaneous signal values obeying the Tikhonov distribution law

$$W_1(x)=\frac{1}{2\pi\times I_0(D)}\exp(D\cos x) \text{ within } -\pi\le x\le\pi. \quad (2.34)$$

The authors of the book [5 p. 46] without additional explanations call the statistical law (2.34) the distribution of V.I. Tikhonov, a well-known scientist who was the first to propose it to describe the phase of self-oscillations of a synchronized generator in a phase-locked loop system. Apparently, we are the leader in the use of this statistical law in the formation of a quasi-deterministic signal.

Similarly to the cosine law (2.20), in the mathematical model (2.34) there are no quantitative parameters of the distribution law, except for the coefficient *D*, which determines the shape of the probability density graph, since it enters the Bessel function I_0(D). Tikhonov's law is centered, its dispersion is determined [15, p.334]

$$\sigma_c^2 = \int_{-\pi}^{\pi} x^2 W_1(x)dx = \frac{\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1} I_n(D)}{n^2 I_0(D)} , \qquad (2.35)$$

it is constant and depends on the coefficient *D*, for example, at *D*=1 the dispersion is equal to $\sigma_c^2 = 1{,}604$,, where $I_n(D)$ is the Bessel function of the imaginary argument of the n^{th} order of the first kind.

The signal amplitude (2.33) is distributed according to the law described by the probability density of the form [16]

$$W_1(y) = \frac{y}{2\pi I_0(D)} + \frac{2y}{\pi I_0(D)}\sum_{n=1}^{\infty} I_n(D) J_0(ny) + \frac{2y}{\pi[I_0(D)]^2}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} I_n(D) I_k(D) J_0\left(y\sqrt{n^2+k^2}\right). \qquad (2.36)$$

Since the statistical law (2.36) was obtained for the first time, we will call it the **Bessel-Lommel law** by analogy with the known functions included in it. The properties of the law (2.36), prescribed in the theory of probability, have been verified by us and they are fulfilled. The Bessel-Lommel law describes the distribution of the random signal amplitude (2.33) within $0 \le y \le \pi$.

The expectation of a random signal amplitude is [16]

$$m_1(a) = \frac{\pi^2}{6I_0(D)} - \frac{2}{\pi I_0(D)}\sum_{k=1}^{\infty}\frac{I_k(D)}{k^3} + \frac{2}{I_9(D)}\sum_{k=1}^{\infty}\frac{I_k(D)}{k^2}\sum_{k=1}^{\infty}\left[J_0(k\pi)S_{1,-1}(k\pi) + J_1(k\pi)S_{2,0}(k\pi)\right] -$$

$$-\frac{2}{\pi[I_0(D)]^2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{I_k(D)I_n(D)}{(k^2+n^2)^{3/2}} + \frac{2}{[I_0(D)]^2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{I_k(D)I_n(D)}{(k^2+n^2)} \times$$

$$\times\left[J_0\left(\pi\sqrt{k^2+n^2}\right)S_{1,-1}\left(\pi\sqrt{k^2+n^2}\right) + J_1\left(\pi\sqrt{k^2+n^2}\right)S_{2,0}\left(\pi\sqrt{k^2+n^2}\right)\right], \qquad (2.37)$$

and the initial moment of the second order of the signal amplitude will be

$$m_2\{a\} = \frac{\pi^3}{8I_0(D)} + \frac{16}{\pi I_0(D)}\sum_{k=1}^{\infty}\frac{I_k(D)}{k^4} + \frac{2}{I_0(D)}\sum_{k=1}^{\infty}\frac{I_k(D)}{k^3}\left[2J_0(k\pi)S_{2,-1}(k\pi) + J_1(k\pi)S_{3,0}(k\pi)\right] +$$

$$+\frac{16}{\pi[I_0(D)]^2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{I_k(D)I_n(D)}{(k^2+n^2)^2} + \frac{2}{[I_0(D)]^2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{I_k(D)I_n(D)}{(k^2+n^2)^{3/2}} \times$$

$$\times\left[2J_0\left(\pi\sqrt{k^2+n^2}\right)S_{2,-1}\left(\pi\sqrt{k^2+n^2}\right) + J_1\left(\pi\sqrt{k^2+n^2}\right)S_{3,0}\left(\pi\sqrt{k^2+n^2}\right)\right]. \qquad (2.38)$$

In this case, the dispersion of the signal amplitude will be $\sigma_a^2 = m_2\{a\} - m_1^2\{a\}$. The designations in expressions (2.37), (2.38) were explained earlier when describing formulas (2.24), (2.25), (2.28), (2.35).

From the analysis of Tikhonov law, it follows that the random phase of the signal (2.33) is distributed according to a uniform law within -π…+π.

The characteristic function of the signal (2.33) is the Fourier transform of the probability density (2.34)

$$\Theta_1(V_m) = \int_{-\infty}^{\infty} W_1(x)\exp(jV_m x)dx = \frac{I_{Vm}(D)}{I_0(D)}. \quad (2.39)$$

Properties of ch.f. depend on properties $I_{Vm}(D)$ - the Bessel function of the imaginary argument V_m - th order of the first kind. The graph of this Bessel function is shown in the figure in the handbook [17, p.196]. For each value of the parameter D, the graph of the function is different, however, for the value $V_m \to \infty$ function $I_{Vm}(D) = 0$. Thus, it can be argued that the properties of the ch.f. (2.39) are observed. If the signal (2.33) has expectation e_0, then its ch.f. will be

$$\Theta_1(V_m) = \frac{I_{Vm}(D)}{I_0(D)}\exp(jV_m e_0). \quad (2.40)$$

Concluding the analysis of the probabilistic characteristics of the quasi-deterministic signal (2.33), let us clarify that its instantaneous values are distributed according to the Tikhonov law, the amplitude - according to the Bessel-Lommel law, and the phase - according to the uniform law.

The signal correlation function (2.33) will be [16]

$$k_u(\tau) = \int_0^{\pi}\int_{-\pi}^{\pi} u(t)\times u(t+\tau)W(y)W(\eta)dyd\eta = \frac{1}{2}m_2\{a\}\cos\omega_0\tau, \quad (2.41)$$

where $W(y)$ is the amplitude probability density (2.36); $W(\eta)$ - probability density of the random phase η; $m_2\{a\}$ - the initial moment of the second order (2.38). Let us proceed to the analysis of the power spectral density (energy spectrum) of signals (2.33). Let's write the energy spectrum of the signal

$$G_u(\omega) = \int_{-\infty}^{\infty} k_u(\tau)\exp(-j\omega\tau)d\tau = \frac{\pi}{2}m_2\{a\}\times[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]. \quad (2.42)$$

The spectrum (2.42) turned out to be lined. It contains a spectral component $\delta(\omega-\omega_0)$ ranging from -∞ to 0 and a spectral component $\delta(\omega-\omega_0)$ ranging from 0 to ∞, where $\delta(\cdot)-$ where delta is a function. In the transition to the physical spectrum, i.e. to the spectrum in the region of positive frequencies, we obtain

$$G_u(\omega) = \pi\times m_2\{a\}\delta(\omega+\omega_0). \quad (2.43)$$

Similarly to what was said earlier, the physical spectrum of a signal with a distribution according to Tikhonov's law contains only one spectral component located on the frequency axis at the point with the abscissa ω_0, when $\omega = 0$ when and coincides with the origin of coordinates.

Completing the stage of formation of mathematical models of quasi-deterministic signals for statistical modulation, let's say that we recorded three of them for the first time with all probabilistic characteristics, including the characteristic function. A quasi-deterministic signal with an arcsine distribution law already exists practically as a source of physical oscillations and can be used when performing statistical modulation. The remaining quasi-deterministic signals can be implemented in practice only in the form of new computer programs, which will later serve as signal sensors as part of digital technologies. Now it is premature to talk about the creation of new sources of physical oscillations, in our opinion. However, the filling of a separate class of random processes with other quasi-deterministic signals must go on constantly.

3. NEW GENERATION MODEMS

Modems with amplitude, phase, frequency modulation are widely used in communication technology, but they have low noise immunity when operating in a noisy channel, and amplitude modulation is the most unprotected from interference. Currently, they are trying to increase the noise immunity of modems by combating interference, while inventing various devices and blocks for suppressing interference, which, at times, are much more complicated than the modems themselves.

We offer another direction for improving the theory of modulation and another way of building modems, based on the complication of the mathematical model of the signal and the modulation of its characteristics, in particular the characteristic function. It is defined in the domain of probabilities or the space of probabilities proposed in 1933 by academician A. Kolmogorov when building information theory. At the same time, the probability theory is a mathematical tool for describing all signal transformations in the probability space. For a signal with a mathematical model (2.1, 2.9, 2.19, 2.33), the characteristic function (ch.f.) is strictly defined, i.e. fundamentally. Thus, by introducing random variables into the models (2.1, 2.9, 2.19, 2.33), a transition to the model of the so-called quasi-deterministic signal, which is an element of statistical radio engineering, is achieved. At first glance, replacing a deterministic oscillation model with a quasi-deterministic signal mathematical model is a fairly simple operation, but the modem noise immunity after replacing the oscillation model turns out to be limiting in the sense that there are no errors when receiving data.

3.1. The first method of signal modulation

We will consider a new modulation method [18], in which all signal parameters are “hidden” inside the expectation operator, as a result of which we obtain the function

$$\Theta(V_m) = m_1\{\exp(jV_m u(t))\}, \tag{3.1}$$

widely known in mathematics, physics, statistical radio engineering. The mathematician A. Lyapunov proposed this function and published its description in 1901 [19]. In the literature [4], it is called the characteristic function. Applying L. Euler's formula, let’s write

$$\Theta(V_m) = m_1\{\cos(V_m u(t))\} + jm_1\{\sin(V_m u(t))\} = A(V_m) + jB(V_m), \tag{3.2}$$

where $A(V_m)$, $B(V_m)$ – real and imaginary parts of the characteristic function; V_m is the parameter of the characteristic function.

By analogy with cosmonautics, the characteristic function (ch.f.) is a “spacesuit” for a signal, it serves as a fundamental probabilistic characteristic of a signal, for example, a quasi-deterministic oscillation (2.1)

$$u(t) = U_0 \sin(\omega_0 t + \eta)$$

with parameters $U_0, \omega_0, \Phi(t) = \omega_0 t + \eta$, where η – a random phase shift angle with a uniform distribution law within − π... + π. The physical meaning of ch.f. studied in [2], and it is shown that it is the spectral density of the probabilities of the instantaneous values of the signal (2.1). Ch.f. depends on the probability density of the signal. Consequently, each model of a quasi-deterministic signal has its own fundamental ch.f., which has many positive properties. It is limited, measurable, filters noise, has limiting values $\Theta(0)=1$, $\Theta(\infty)=0$, $\Theta(-\infty)=0$. Other remarkable properties of it are described in [2]. Based on the advantages of the ch.f., we propose a method for modulating this function.

A ch.f. modulation method in which a constant voltage is multiplied with a telegraph signal *s(t)*, which takes on the value either "1" or "0", after which the product $e_0 s(t)$ is summed with a centered quasi-deterministic signal (2.1), expectation which is equal to zero, and thus carry out the modulation of the ch.f. of the transformed quasi-deterministic signal according to the law:
for *s(t)*=0 to obtain functions of the form

$$A(V_m,t) = J_0(V_m U_0,t), \qquad B(V_m,t) = 0\ ; \tag{3.3}$$

for *s(t)*=1 to obtain functions of the form

$$A(V_m,t) = J_0(V_m U_0,t) \cos(V_m e_0),\ B(V_m,t) = J_0(V_m U_0,t) \sin(V_m e_0), \tag{3.4}$$

where $J_0(\cdot)$ is Bessel function of zero order; U_0 – the signal amplitude V_m – the ch.f. parameter, and at $V_m = 1$ function $A(1,t)$ and function $B(1,t)$ change in antiphase. By the way, the dependence of the ch.f. from time to time appeared due to the modulation of the signal, since the modulated signal is a non-stationary process.

In the future, we propose to call the modulation of a new type **statistical modulation** (SSK - statistical shift keying).

A block diagram of the modulator is shown in Figure 3.1, it contains a multiplier 1 and an adder 2. Timing diagrams explaining its operation are shown in Figure 3.2. The following explanations can be given to the figures. In accordance with the definition of the modulation method, a non-centered quasi-deterministic signal is formed

$$u_1(t) = e_0 s(t) + U_0 \sin(\omega_0 t + \eta) \tag{3.5}$$

with ch.f. as [2]

$$\Theta(V_m,t) = J_0(V_m U_0,t) \exp(jV_m e_0). \tag{3.6}$$

Let the telegraph signal be a sequence of logical zeros and ones (Fig. 3.2a). If *s(t)*=0, then the ch.f. has only a real part, and its imaginary part is equal to zero [2], i.e.

$$\Theta(V_m,t) = A(V_m,t) = J_0(V_m U_0,t), B(V_m,t) = 0.$$

In this case, with V_m=1, we have A (1, t), B (1, t) in Figure 3.2d, e. When $s(t)$=1, the ch.f. is equal to (3.6). Then we get

$$A(V_m,t) = J_0(V_m U_0,t) \cos (V_m e_0), \quad B(V_m,t) = J_0(V_m U_0,t) \sin (V_m e_0).$$

At V_m=1 we have functions

$$A(1,t) = J_0(U_0,t) \cos (e_0), \quad B(1,t) = J_0(U_0,t) \sin (e_0), \tag{3.7}$$

which are shown in Figure 3.2d, e. These functions change according to the law of the telegraph signal. Therefore, ch.f. modulated by a telegraph signal, and the functions A(1, t), B(1, t) change in antiphase.

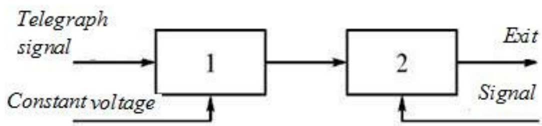
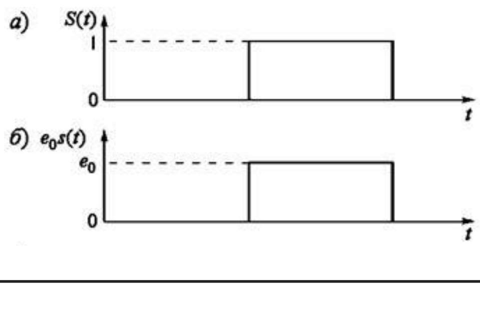


Figure 3.1. *Signal modulator circuit*

In our opinion, the structure of the modulator turned out to be simple; there are no complex nodes and sources of oscillations in it. Quasi-deterministic signal (2.1) is present at the output of any self-oscillator up to atomic frequency standards. It is known from the review [20] that they and quartz oscillators have short-term phase instability, or, in other words, phase fluctuations, and thus fall under the signal model (2.1). Moreover, the characteristics in Figures 2.1 - 2.4, measured experimentally when studying the signal at the output of a standard generator, confirm this. Constant voltage value e_0 by analogy with computers, cell phones can be obtained from the battery. In addition, the amplitude of the high-frequency oscillation in the modulator does not change and, as a result of this, the power amplifier of the transmitting device has the linearity of the input and output characteristics in a narrow range of input signals. All taken together characterizes the modulator only from the positive sides.



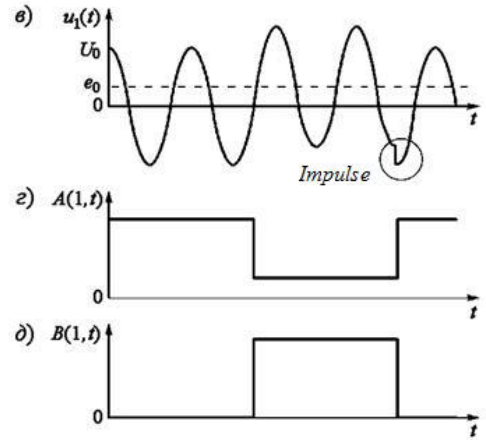


Figure 3.2. *Timing waveform diagram in the modulator*

However, this modulation method, in which there is a constant component of the signal, is used only in wired communication and is not used in radio communication. Antenna-feeder devices (AFD) in radio communications do not pass the constant component of the signal. For radio communications, a method for amplitude shift keying (AM) of a signal has been developed, which can be used in statistical modulation, since ch.f. of a quasi-deterministic signal will change, similarly to the amplitude of a deterministic oscillation. In this case, the amplitude-time diagram in Figure 3.2c will be different and will take the form shown in Figure 3.2e. The power amplifier of the transmitting device will change the linear mode of operation to non-linear.

At the *AM* of a centered quasi-deterministic signal (2.1)

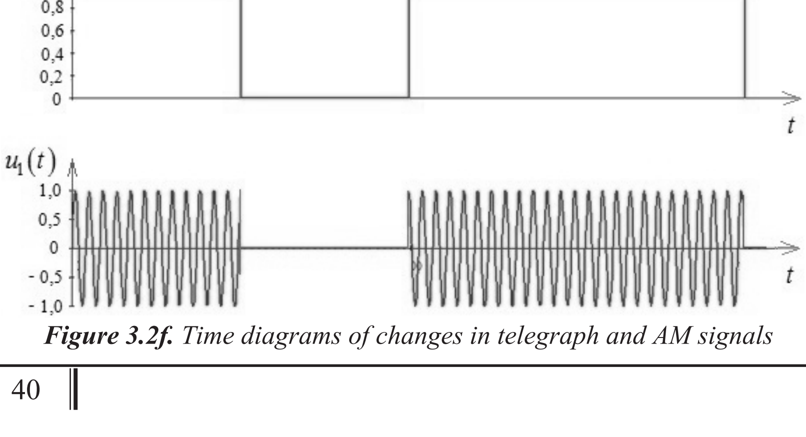


Figure 3.2f. *Time diagrams of changes in telegraph and AM signals*

We obtain only the real part of the ch.f., which changes in antiphase with the telegraph signal and has the form shown in Figure 3.2d. The imaginary part of the ch.f. centered signal (2.1) is always zero. Schemes of modulators with amplitude keying of the signal are described in detail in textbooks [21] and monographs [22].

In passing, we present in more detail the diagram in Figure 3.2c for the telegraph signal shown in Figure 3.2e. As a result, we get a non-centered quasi-deterministic signal with amplitude keying, shown in Figure 3.2g.

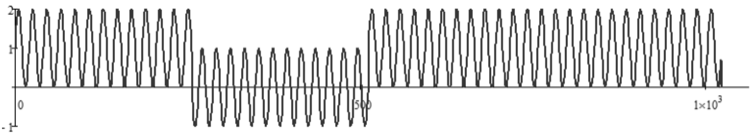


Figure 3.2g. *Non-centered amplitude-shift keying signal*

3.2. The second method of signal modulation

Let a quasi-deterministic signal (2.9) be modulated, for which the random variable *a* is distributed according to the normal law with quantitative parameters - expectation, e_0 - expectation, σ^2 - dispersion, and the random η variable is distributed uniformly within 0…2π. Then we have

$$u_2(t) = [s(t) \times a] \sin(\omega_0 t + \eta). \quad (3.8)$$

The characteristic function of A. Lyapunov for the quasi-deterministic signal (2.9), obtained by us earlier using the well-known expression [4, p.263]

$$\Theta(V_m) = \int_0^\infty J_0(xV_m) W_1(x) dx \quad (3.9)$$

and tables [11] , taking into account the distribution law of the random variable *a* at $V_m > 0$, $e_0 = 0$ has the form (2.12). Ch.f. has properties [2], for example, if the law W_1(x) corresponds to the ch.f. $\Theta(V_m)$, then the law $W_1(x \pm e_0)$ corresponds to the ch.f. $\Theta(V_m)\exp(\mp jV_m e_0)$. Therefore, for a law with expectation $e_0 \neq 0$ ch.f. (2.12) is transformed into the expression

$$\Theta(V_m) = m_1\{jV_m u_2(t)\} = I_0\left(\frac{V_m^2\sigma^2}{4}\right)\exp\left(-\frac{V_m^2\sigma^2}{4}\right)\exp(jV_m e_0). \quad (3.10)$$

Ch.f. (3.10) at V_m=const, excluding zero and infinity, depends on the variables e_0, σ^2. Consequently, by changing the expectation and amplitude dispersion of the quasi-deterministic signal (2.9) with the help of a telegraph signal *s(t)*, one can modulate the ch.f. this signal. Using this method of influencing the amplitude, it is possible to implement twelve variants of the considered method of ch.f. modulation. signal. A new method, the so-called statistical modulation, using a characteristic function, a quasi-deterministic signal $u_2(t)$ and a telegraph message *s(t)*, consists in changing the quantitative parameters of the distribution law of the amplitude of the quasi-deterministic signal in accordance with the change in the telegraph message containing a sequence of logical "0" and logical "1".

A block diagram of the modulator is shown in Figure 3.3 [13], it contains a (IC) interface circuit, a centered quasi-deterministic signal sensor (c.q.s) and a (SB) settings block. In the settings block, the values of the quantitative parameters of the distribution law of the amplitude of the quasi-deterministic signal are stored in the memory, which are written to the signal sensor through the interface circuit. The algorithm for writing parameters includes a telegraph signal *s(t)*, from the logical "0" and "1" of which the values of the settings depend. For example, when a logical "0" arrives, the setting σ_0^2 =1, and when a logical «1» arrives - σ_1^2= 0,0009 is selected, while in both cases the setting e_0=0. Then at the output of the modulator we get a modulated centered quasi-deterministic signal, which is shown in Figure 3.4. In form, the time diagram of the signal resembles a random process that obeys the statistical law of Veshkurtsev, with a probability density of the form (2.11).

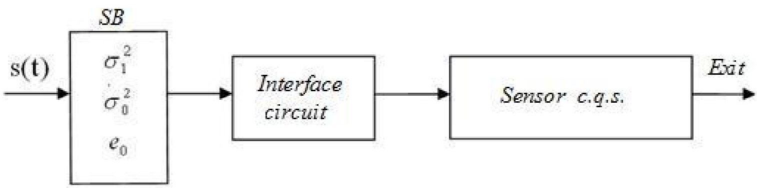


Figure 3.3. *Modulator circuit*

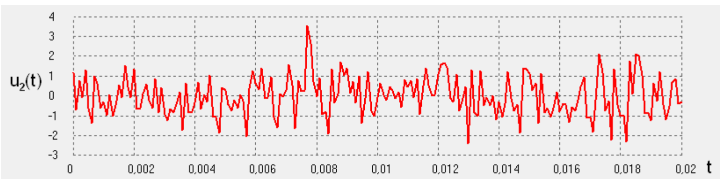


Figure 3.4. *Timing diagram of the modulated signal*

In the modulator circuit in Figure 3.3, there is no source of physical oscillations, and instead of it, a centered quasi-deterministic signal sensor (c.q.s.) is used. This sensor is built as a computer program using digital technology. The same can be said about other blocks of the modulator block diagram, which are separate files of the general program.

The instantaneous values of the centered quasi-deterministic signal (2.9) vary in the range ±3σ and can reach large values at σ=1, where σ – is the mean square deviation (MSD) of the signal amplitude. This, in turn, places great demands on the linearity of the input and output characteristics of the power amplifier of the transmitting device.

3.3. Combined signal modulator

The first and second methods of signal modulation are implemented individually using their own modulator. However, it is possible to build some combination of two modulators [16], shown in Figure 3.5. The combined modulator circuit includes a multiplier 1, an adder 2, a setpoint block 3, a generator or sensor of a centered quasi-deterministic signal 4. In contrast to the modulator in Figure 3.3, the setpoint block contains only the energy quantitative parameters of the generator (or sensor) oscillation in the form of dispersions σ_c^2, σ_0^2, σ_1^2 signal, which are set using the logical "1" and "0" of the telegraph signal.

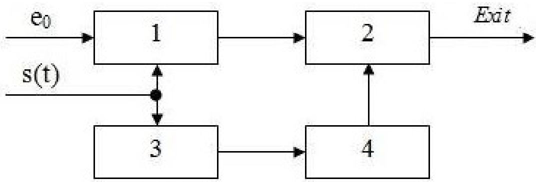
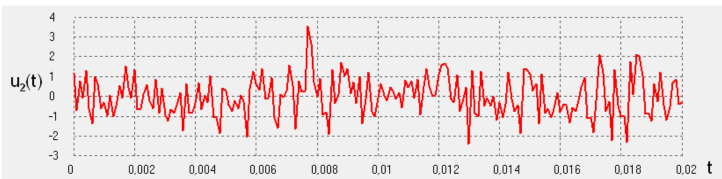
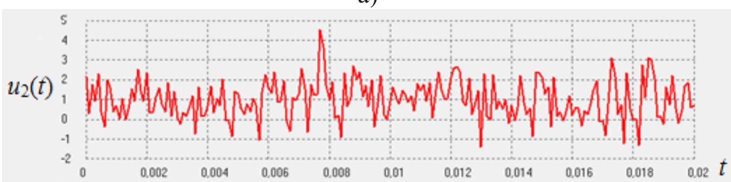


Figure 3.5. *Modulator circuit*

The expectation e_0 is introduced using a telegraph signal *s(t)* through another channel containing a multiplier 1 and an adder 2, which receives a centered quasi-deterministic signal from a sensor (or generator). As a result of these transformations, at the output of the modulator, we obtain a non-centered quasi-deterministic signal shown in Figure 3.6.



a)



b)

Figure 3.6. *Timing diagrams of the modulated signal*

The amplitude-time dependence of the fluctuation in Figure 3.6a does not contain the expectation (e_0 =0), while in Figure 3.6b the quasi-deterministic signal (2.9) has the expectation e_0 =1.
As a result of this, the characteristic function of the signal (2.9) will be modulated, and the variance $\sigma_0^2 = \sigma_1^2$ of the signal (2.9) in both pictures is constant, where σ_0^2, σ_1^2 is the variance of the signal amplitude (2.9) when a logical “0” and a logical “1” arrive at the modulator telegraph signal, respectively.

The amplitude-time dependence of the oscillation at the output of the modulator with a non-centered quasi-deterministic signal (3.5) is shown in Figure 3.2c. Let's recall that the adder 2 of the modulator in this case receives a quasi-deterministic signal (2.1) with a constant dispersion σ_c^2 from generator 4, which in this case replaces the sensor.

3.4. Two-channel signal demodulator

To demodulate the signal, we propose a new method [23], which uses an analog-to-digital signal conversion, multiplication of discrete instantaneous signal values with the parameter V_m, a functional transformation in order to obtain the functions of the sine and cosine products, followed by the accumulation of the values of these functions over a time interval equal to the duration symbol logical "0" and logical "1". After that, using the sine function, the estimate $\widehat{B}(V_m, t)$ of the imaginary part of the ch.f. is calculated, and using the cosine function, the estimate $\widehat{A}(V_m, t)$ of the real part of ch.f., the current values of which are compared with the thresholds, and the decision is made in accordance with the fulfillment of the following inequalities:

1) if $\widehat{B}(V_m, t) < \Pi_{1c}$, then it is considered that the logical „0" is accepted;

2) if $\widehat{B}(V_m, t) \geq \Pi_{1c}$, then it is considered that the logical „1" is accepted;

3) if $\widehat{A}(V_m, t) \geq \Pi_{2к}$, then it is considered that the logical „0" is accepted;

4) if $\widehat{A}(V_m, t) < \Pi_{2к}$, then it is considered that the logical „1" is accepted.

The block diagram of the demodulator is shown in Figure 3.7. It contains an analog-to-digital converter (ADC) 1, a multiplier 2, functional converters 3,4, accumulating averaging adders 5,6, threshold devices 7,8, an inverter 9. The principle of operation of the demodulator is as follows. The demodulator input receives, for example, a signal (3.8). After conversion to the ADC, the discrete instantaneous values of the signal u_2 $(k\Delta t)$ are multiplied with the parameter V_m, and the products are converted to obtain the function sin $[u_2 (k\Delta t)V_m]$ and the function cos $[V_m u_2 (k\Delta t)]$. Accumulating averaging adders 5.6 work simultaneously. The adder 5 accumulates the current values of the sine function, and the adder 6 - the current values of the cosine function. When a synchronization pulse appears at the strobe inputs of the adders, the estimates of the real and imaginary parts of the ch.f. appear at their outputs.

$$\hat{A}(V_m, t) = \frac{1}{N}\sum_{k=1}^{N}\cos\,[V_m u_2(k\Delta t)], \qquad (3.11)$$

$$\hat{B}(V_m, t) = \frac{1}{N}\sum_{k=1}^{N}\sin\,[V_m u_2(k\Delta t)], \qquad (3.12)$$

where N - sample size of instantaneous signal values; Δt is the signal sampling interval. The properties of the estimates (3.11,3.12) were studied in [2], and it was found that for N>>1 they are asymptotically consistent, effective, and unbiased.

The values of the estimates of the ch.f. (3.11,3.12) with the value V_m=1 are compared in threshold devices 7,8 with the threshold Π_{1c}, Π_{2k}. For convenience of analysis, the series connection of blocks 3, 5, 7 will be called the **sine channel** of the demodulator, and the series connection of blocks 4, 6, 8, 9 will be called the **cosine channel** of the demodulator. Each channel has its own output, hence the demodulator has two outputs. At the output of the cosine channel, the telegraph signal is received inverse with respect to the original. Therefore, the inverter 9 is turned on at the channel output. If the above inequalities with the value V_m=1 are not met, errors occur in the decision regarding the received symbol of the telegraph signal.

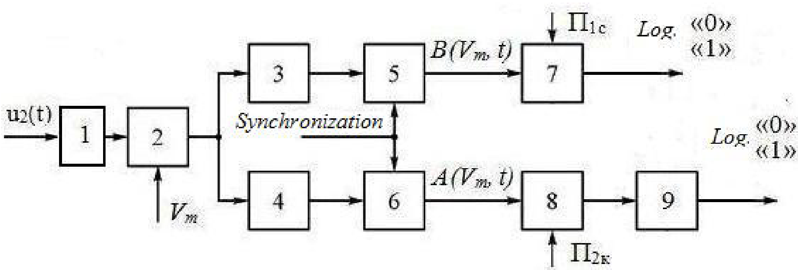


Figure 3.7. *Two-channel signal demodulator*

Thresholds Π_{1c} (sine channel), Π_{2k} (cosine channel) are set in accordance with the equalities

$$\Pi_{1c} = K_1\Pi_1,\ \Pi_{2k} = K_2\Pi_2, \qquad (3.13)$$

where K_1, K_2 - variable coefficients; Π_1, Π_2 are thresholds, the values of which will be different depending on the model of the quasi-deterministic signal and are calculated further when analyzing the noise immunity of the modem.

3.5. Single-channel signal demodulator

The sine and cosine channels of the demodulator in Figure 3.7 are not equally affected by interference and, as a result, have different noise immunity. If we combine the advantages of each channel together, we get a single-channel demodulator circuit [24], shown in Figure 3.8.

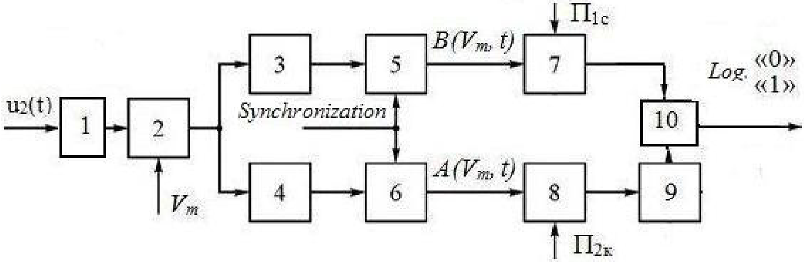


Figure 3.8. *Structural diagram of the demodulator*

The block diagram of the demodulator is shown in Figure 3.8. It contains an analog-to-digital converter (ADC) 1, a multiplier 2, functional converters 3.4, accumulating averaging adders 5.6, threshold devices 7.8, an inverter 9, logical AND circuit. Logic circuit 10 combines the outputs of the demodulator channels in the figure 3.7, after which the demodulator has only one output. The demodulator becomes a single-channel device.

Up to logic diagram 10, the single-channel demodulator operates in full accordance with the description of the principle of operation of the circuit in Figure 3.7. Further, the logic circuit 10 is included in the work, the operation of which is explained in Table 3.1.

Table 3.1.
Truth or state table

Sequence number	1	2	3	4
Sinus channel output	log. «1»	log. «1»	log. «0»	log. «0»
Cosine channel output	log. «1»	log. «0»	log. «1»	log. «0»
Demodulator output	log. «1»	log. «0»	log. «0»	log. «0»

Looking ahead, let's say that in the first case, when determining the logical "1", errors are possible, since the sinus channel determines the logical "1" satisfactorily. But the logical "0" sinus channel determines without errors. Therefore, in all subsequent cases, the absence of errors can be expected. The simulation of the circuit in Figure 3.8 confirms what has been said [24]. On average, the single-channel demodulator in Figure 3.8 reduces the error rate by a factor of 20 compared to the cosine channel of the two-channel demodulator. The circuit in Figure 3.8 is not the only one; other options for combining the demodulator channels in Figure 3.7 are possible. Additional studies are required to determine the optimal option for combining two demodulator channels into one channel.

Consider another version of the single-channel demodulator [25], shown in Figure 3.9. In fact, there is actually only one channel in it, namely, this is the previously designated cosine channel. However, the circuit in Figure 3.9 can equally belong to the sine channel of the demodulator if the FC will form a sine function.

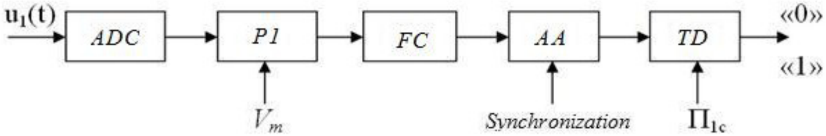


Figure 3.9. *Single-channel signal demodulator*

The circuit in Figure 3.9 includes an ADC - an analog-to-digital converter; P1 - multiplier; FC is the functional converter of the cosine function; AA - accumulative averaging adder; TD is a threshold device, in the output circuit of which an inverter is included, as is done, for example, in the circuit of Fig. 3.7. Therefore, at the output of the demodulator, we will receive an inverse set of logical "0" and "1", which are taken from the output of the control panel to the inverter.

Signal conversion, for example (3.8), in the demodulator proceeds in the following sequence. The quasi-deterministic signal (3.8) is discretized by the ADC, and each discrete instantaneous value of the signal $u_2(k\Delta t)$ is multiplied with the ch.f. parameter $V_m u_2(k\Delta t)$, the product is converted by the functional converter into the value of the function $\cos[V_m u_2(k\Delta t)]$, where Δt is the sampling interval of the signal. The values of the cosine function are accumulated in the adder, and when a synchronization command is received, they are averaged. The result of averaging enters the threshold device and is compared with the threshold, and the decision is made in accordance with the inequalities:

1) if $\widehat{A}(V_m, t) \geq \Pi_{1c}$, then it is considered that the logical „1" is accepted;

2) if $\widehat{A}(V_m, t) < \Pi_{1c}$, then it is considered that the logical „0" is accepted.

The result after averaging the AA adder data is

$$\widehat{A}(V_m, t) = \frac{1}{N}\sum_{k=1}^{N}\cos[V_m u_2(k\Delta t)], \tag{3.14}$$

where N is the sample size of discrete instantaneous values of the signal. In expression (3.14), the expectation operator is replaced by an ideal adder. Studies of the evaluation of the real part of the ch.f. showed [2] that, as $N \to \infty$ it is asymptotically consistent, efficient, and unbiased, i.e., evaluation properties tend to fundamental properties. Consequently, the value of the estimate (3.14) will be equal to the value of the ch.f. (2.12), while the threshold will be (3.13)

$$\Pi_{1c} = K_1 \Pi_1$$

where K_1 - variable coefficient; Π_1 – TD device threshold. The coefficient K_1 in each modem is different, it depends on the signal modulation algorithm.

4. NOISE IMMUNITY OF THE MODEM IN THE CHANNEL WITH "WHITE" NOISE

The theoretical analysis of modem noise immunity is based on the determination of the real and imaginary parts of the ch.f. additive mixture. Then the values are calculated separately for each of the parts of the ch.f. additive mixture. And finally, these values of the ch.f. are compared with the thresholds set in the sine and cosine channels of the demodulator in order to make decisions in accordance with the observance of the inequalities recorded in the signal discrimination algorithm. Separately, a quantitative analysis of the probability of errors is carried out. In total, thirteen different modems of the new generation are considered together. To determine whether the material in this chapter belongs to the device model, the modem name was used, which includes a cipher of letters and numbers denoting the following: A - arcsine law; K is the law of cosine; B - Veshkurtsev's law; T is Tikhonov's law; 1 - one channel; 2 - two channels; 2-1 - one channel resulting from combining two different demodulator channels using digital logic circuits. Let's correctly write down and decipher, for example, such a name: A2-1 modem is a single-channel modem for receiving signals with distribution according to the arcsine law.

4.1. Noise immunity of modem A when receiving an additive mixture of noise and signal with the distribution of instantaneous values according to the arcsine law

Let's recall that a signal with the distribution of instantaneous values according to the arcsine law can be modulated in two ways, described above in Section 3.1. We will consider each of them separately and, on their basis, we will build modems that are different in structure and characteristics. As a result, we get three models of a new generation modem.

4.1.1. Noise immunity of the modem A2 when receiving an additive mixture of noise and a non-centered signal with the distribution of instantaneous values according to the arcsine law

The modem contains a modulator (Fig. 3.1), the quasi-deterministic signal at the output of which is shown in Fig. 3g, and a two-channel demodulator (Fig. 3.7). Its name will be: **modem A2**. The modulation algorithm for a quasi-deterministic signal (2.1) is written in Table 4.1.

Table 4.1.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,18	0
logical "1"	0,18	0,9

The methodology and results of the studies were published in [26]. Let us turn to the analysis of the noise immunity of the demodulator under the action of an additive mixture of a quasi-deterministic signal (2.1) and "white" noise at its input

$$z(t)=u(t)+n(t), \tag{4.1}$$

where $n(t)$ is "white" noise, $u(t)$ is a signal with $a=U_0$, and the probabilistic characteristics are known from section 2.1.

Using expressions (3.3, 3.4) and the data in Table 4.1, using formulas (3.13), we calculate the thresholds in the sine and cosine channels of the demodulator. As a result, with the value V_m = 1 and U_0 =0,6 we get

$$П_1=J_0(U_0,t)\sin(e_0) = 0,7116; \quad П_2 = J_0(U_0,t) = 0,912.$$

Further, with the value V_m=1 and $s(t)$ =0 we define for the additive mixture (4.1)

$$A(1,t) = \int_{-\infty}^{\infty} \cos(z)W(z)dz = J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right) =$$

$$= J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right). \tag{4.2}$$

When $s(t)$ = 0, similarly to (4.2), we calculate for the value V_m = 1 for the additive mixture (4.1)

$$B(1,t) = \int_{-\infty}^{\infty} \sin(z)W(z)dz = 0, \tag{4.3}$$

where W(z) – probability density of instantaneous values of the additive mixture; $h = {}^{\sigma_c}\!/_{\sigma_ш}$ – signal-to-noise ratio; $\sigma_c^2 = {}^{U_0^2}\!/_2$ – the dispersion of the quasi-deterministic signal; $\sigma^2_ш$ – the dispersion of "white" noise. The results (4.2, 4.3) need to be quantified. Tables 4.2, 4.3 present the results of calculations for $П_1$=0,7116; $П_2$=0,912; K_1=0,56; K_2=0,88, written in the line with the name of the evaluation.

Table 4.2.

The probability of errors in the cosine channel at a logical "1"

Threshold $П_{2k}$	0,912· 0,88 = 0,8					
Evaluation $\hat{A}(1,t)$	0	0	0,37	0,83	0,9	0,9
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$2{,}2·10^{-5}$	$2·10^{-45}$	$2·10^{-45}$

Table 4.3.

Probability of errors in the sinus channel at logical "0"

Threshold $П_{1c}$	0,7116· 0,56 = 0,4					
Evaluation $\hat{B}(1,t)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	0	0	0

When analyzing the data in tables 4.2, 4.3, we always compare the values of the estimates of ch.f. additive mixture with the thresholds recorded in the first line of the tables. At the same time, we see that the data in Table 4.2 exceeds the threshold, starting from the signal-to-noise ratio from 1 to 100, i.e. in the range of 20 dB. This means that there will be no errors here when receiving a logical "0", so the modem has maximum noise immunity. In the range of signal-to-noise ratios from 0.1 to 1, errors when receiving a logical "0" are possible. However, it can be stated that the noise immunity of the cosine channel of the modem is an order of magnitude better than the data given in the publication. Analyzing the data in Table 4.3, we see ideal results. In the sinus channel of the demodulator, all data is below the set threshold. Therefore, we have the maximum noise immunity when receiving a logical "0" in the range of signal-to-noise ratios from 10^{-3} to 10^{2} or 50 dB, and the lower limit of the range is minus 30 dB. These data are at least twenty orders of magnitude better than the noise immunity of the device known from the publication.

Suppose the additive mixture (4.1) contain a non-centered quasi-deterministic signal at the demodulator input; this corresponds to the condition *s(t)*=1. Similarly, to (4.2), for the value V_m=1 we define

$$\begin{aligned} A(1,t) &= \int_{-\infty}^{\infty} \cos(z) W(z-e_0)dz = J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\cos(e_0) = \\ &= J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(e_0) \end{aligned} \quad (4.4)$$

or similarly to (4.3) for the value V_m= 1 we calculate

$$B(1,t) = \int_{-\infty}^{\infty} \sin(z) W(z - e_0) dz = J_0(U_0) \exp\left(-\frac{\sigma_ш^2}{2}\right) \sin(e_0) = \\ = J_0\left(\sigma_c \sqrt{2}\right) \exp\left(-\frac{\sigma_c^2}{2h^2}\right) \sin(e_0). \tag{4.5}$$

The results (4.4), (4.5) need a quantitative analysis. Tables 4.4, 4.5 show calculation data at $П_1$=0,7116; $П_2$=0,912; $К_1$=0,56; $К_2$=0,88, written in the line with the name of the evaluation.

Table 4.4.

Probability of errors in the cosine channel at logical "0"

Threshold $П_{2k}$	0,912· 0,88 = 0,8					
Evaluation $\widehat{A}(1,t)$	0	0	0,23	0,52	0,57	0,57
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$

Table 4.5.

The probability of errors in the sinus channel at a logical "1"

Threshold $П_{1c}$	0,7116· 0,56 = 0,4					
Evaluation $\widehat{B}(1,t)$	0	0	0,29	0,65	0,71	0,71
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$8{\cdot}10^{-32}$	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$

Similarly, to the analysis of tables 4.2, 4.3, we will study the data of tables 4.4, 4.5. The data in Table 4.4 is below the set threshold. Hence, they correspond to the ideal case. Here we can say that the reception of the logical "1" in the cosine channel of the demodulator occurs without errors, i.e. with ultimate noise immunity, in the range of signal-to-noise power ratios from 10^{-3} to 10^2 or in the range of 50dB. These data are at least twenty orders of magnitude better than the noise immunity of the device known from the publication. The data in Table 4.5 are much more modest than the previous ones. In the sinus channel of the demodulator, a logical "1" is received without errors only when the signal-to-noise ratio is from 1 to 100 or in the range of 20 dB. With a signal-to-noise ratio from 0.1 to 1 in the sinus channel of the demodulator, errors are possible when receiving a logical "1".

Let's move on from qualitative data analysis to a quantitative assessment of modem noise immunity. In tables 4.2–4.5, the following designations are adopted: P_0 – the probability of errors when receiving a logical "0"; P_1 – the probability of errors when receiving a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - is the total probability of device errors.

Quantitative assessment of the noise immunity of the modem A2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 4.2 - 4.5. Both estimates are random variables with their own properties and distribution laws. Let us recall that estimates for the real and imaginary parts of the ch.f. are efficient, consistent, and unbiased. This is shown in earlier works, for example [2], in which the effectiveness of estimates is characterized by their variances. In the book [2, p. 95 – 96] the dependence of the variance of estimates (3.11, 3.12) on the dimensionless time is shown $S = T \cdot \Delta F_э$, where T - the duration of the signal realization; $\Delta F_Э$ - the width of the energy spectrum of the signal. With a value of S =100 the variance of the estimate of the real part of the ch.f. $\sigma_A^2 = 10^{-4}$, and the variance of the estimate of the imaginary part of the ch.f. $\sigma_B^2 = 10^{-3}$. The value S =100 will be obtained when we take with $T = N \cdot \Delta t = 10^3 \cdot 0{,}1мс = 0{,}1s$ and $\Delta F_э = 1000 Гц$. Here the designations are borrowed from expressions (3.11,3.12).

The law of distribution of estimates of the real and imaginary parts of the ch.f. depends on the probability density of the additive mixture of signal and noise. Let it be normal in the first approximation, since it is difficult to solve this problem mathematically exactly, and maybe even not possible. According to Professor S.Ya. Vilenkin, who has been solving similar problems for many decades, "..an exact solution is possible only in some cases [27, p.106]". For example, in the same place, the author obtained the exact distribution law for the estimate of the correlation function of a Gaussian signal, and then, after some assumptions, suggested that it be considered approximately normal. Let's follow this example. Looking ahead, we say that when modeling a demodulator (Fig. 3.7), the validity of such a hypothesis was proved in [28].

Next, we proceed similarly to the procedure for discretizing a continuous value by level, with one level equal to the threshold, and the second level is not limited by the threshold, i.e. it is variable without negative consequences for the probability of errors. At the same time, we consider that the center of the distribution law coincides with the value of the ch.f., recorded in tables 4.2 - 4.5, since estimates of ch.f. are not displaced. There is a corridor between the value of the assessment and the threshold, it is different when h^2. If the value of the estimate of the ch.f. goes beyond the corridor boundary, then an error occurs when receiving a logical element. For example, the corridor is 0.23 in table 4.4 with a value h^2 =10. We divide the value of the corridor by "sigma", i.e. on σ_B estimates of the real or on estimates of the imaginary part of the ch.f. depending on the demodulator channel in question. The mean square value of the estimate in the cosine channel is σ_A = 0,01, so we get the number of "23sigma" separating these two values. Then we apply a rule similar to the “three sigma” rule and calculate the value of the error integral at “*L* sigma”. In our example *L*=23. The error probability that interests us will be equal to the difference between unity and the value of the error integral. Unfortunately, in reference books on special functions [17], the values of the error integral are limited to the size L≤10. Therefore, in tables 4.2 - 4.5, the values of the error probability are sometimes overestimated, for example, in table 4.4 at h^2 =10. In fact, the errors will be smaller by many orders of magnitude.

For greater clarity and understanding of what was said above, we use the well-known distribution law for the estimate of the real part of the ch.f. [28], the view of which is shown in Figure 4.1. We will calculate the modem error probabilities using a method developed on the basis of the statistical decision theory [29]. The estimate is a random variable, depends on the signal-to-noise ratio and has a dispersion σ^2_A of the real part of the ch.f. and the dispersion σ^2_B of the estimate of the imaginary part of the ch.f. The values of estimates (3.11, 3.12) are distributed according to the Gauss law [28], in which the values recorded in tables 4.2 - 4.5 are the most probable, i.e. expectations. The distribution law and the initial data, where $W(A)$ – the probability density of the ch.f. estimate is shown in Figure 4.1a; $m_1\{A\}$ – the expectation of the estimate of the ch.f. You can also see the interval $L_A = |m_1\{A\} - \Pi_{2k}|$ between the mathematical expectation of the estimate and the threshold in the demodulator. In Figure 4.1a, the thresholds are shown to the left and right of the expected value. This is done because either the left or the right half of the distribution law is involved in determining the logical "0" and logical "1".

Let's start calculating the error probabilities in the demodulator when a logical "0" is received. The mathematical expectation of the estimate is equal to the value (4.4). All values of the evaluation (3.11) must exceed the threshold Π_{2k}, shown in Figure 4.1a on the left. Let's use the three sigma rule. Let's define the number of sigmas using the relation L_A/σ_A. The probability of errors when receiving a logical "0" is equal to the area under the curve $W(A)$, lying to the left of the threshold Π_{2k}.

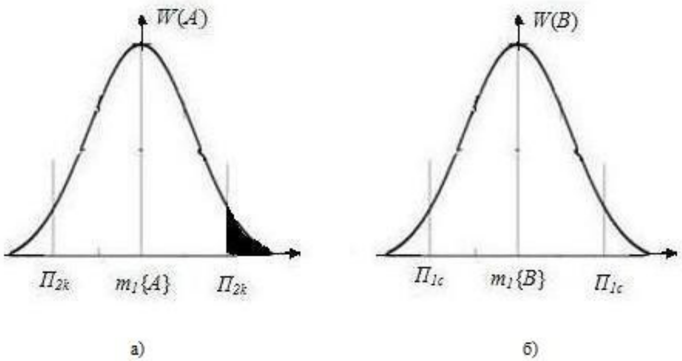


Figure 4.1. *The probability density of the real a) and imaginary b) parts of the ch.f.*

It is equal numerically

$$P_0 = \frac{1}{2}\left[1 - erf\left(\frac{L_A}{\sigma_A}\right)\right] = \frac{1}{2} erfc\left(\frac{L_A}{\sigma_A}\right), \qquad (4.5d)$$

where $erf(\cdot)$, $erfc(\cdot)$ – probability integral (error function). Formula (4.5d) is suitable for calculating errors in the demodulator when a logical "1" is accepted. Only the expectation of the estimate in this case is equal to the value (4.2), and the probability of errors is equal to the area under the curve $W(A)$, which lies to the right of the threshold Π_{2k} (colored in black), and will be denoted by P_1. Then $P = \frac{1}{2}(P_0 + P_1)$ the probability of modem errors. When calculating the error probability according to formula (4.5d), data from tables 4.2, 4.4 were substituted in place $m_1\{A\}$. A similar description can be repeated for estimating (3.12) the imaginary part of the ch.f. using the data in tables 4.3, 4.5 and figure 4.1b.

The total error probability of the sine (curve 1) and cosine (curve 2) demodulator channels is shown in Figure 4.2, and its main values are listed in Table 4.6. For comparison, in the same place from [15, p.473], the probability of errors (curve 3) of ideal phase modulation (PM), calculated in a noisy channel, is given. The choice of PM for comparison is not accidental. It is recognized by all as the most noise-resistant modulation.

Table 4.6.

Probability of errors of different modems

Total sinus channel error probability	0,5	$2 \cdot 10^{-21}$	$4 \cdot 10^{-32}$	$1 \cdot 10^{-45}$	Less than $1 \cdot 10^{-45}$
Total cosine channel error Probability	0,5	$4{,}9 \cdot 10^{-1}$	$1{,}1 \cdot 10^{-5}$	$1 \cdot 10^{-45}$	Less than $1 \cdot 10^{-45}$
PM error probability	0,9	$3{,}2 \cdot 10^{-1}$	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$
Signal-to-noise ratio	0,1	0,5	1,0	10	100

Comparison of the noise immunity of the new modem with the noise immunity of the known device, in which ideal PM is used, shows its superiority by at least four orders of magnitude and more, up to thirty orders of magnitude when working with weak signals. This causes distrust among modem developers, whose opinion says: "this cannot be, because it can never be." In the cosine channel (curve 2), the new modem has a reference point with a non-zero error probability $P = 1{,}1 \cdot 10^{-5}$, i.e. from the limiting noise immunity of the device, if the probability of errors 10^{-32} is conventionally equated to zero. Its occurrence may be associated with a random, without any justification, choice for modulating the quantitative parameters of the distribution law of a quasi-deterministic signal. Probably, the optimization of these parameters will eliminate the modem reference point. There are no reference points in the sinus channel (curve 1) of the modem. Therefore, even with such data, one can hope for a good future for the new modem with two channels.

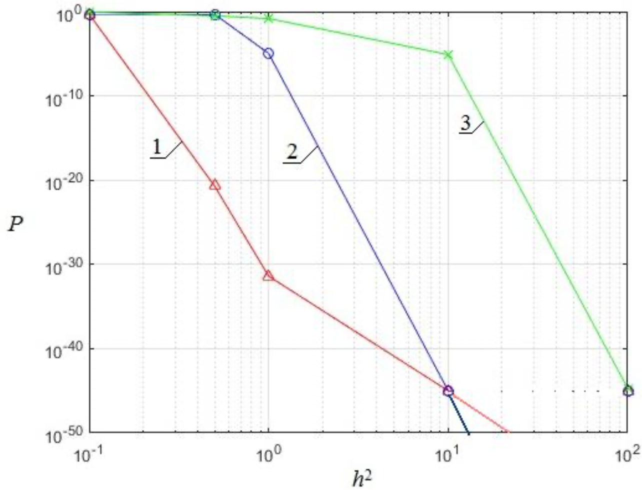


Figure 4.2. *Probability of errors of the two-channel modem A2*

As a result, we can say that in the presence of "white" noise in the data transmission channel, the potential noise immunity according to Kotelnikov of the proposed modem is limiting, because with accurate synchronization of both channels of the modem, there are no errors when receiving a telegraph signal. In the sine and cosine channels of the modem, the range of signal-to-noise power ratios is different. In the sine channel it is equal to 30 dB, and in the cosine channel - 25 dB, and the lower limit of the range in the sine channel lies at the level of minus 10 dB, while in the cosine channel it is equal to minus 5 dB. Thus, the sine channel of the modem has better noise immunity than the cosine channel. In Figure 4.2, curves 2 and 3 run parallel in the section $1 \le h^2 \le 100$. Therefore, the cosine channel at the error probability level of $1 \cdot 10^{-5}$ and less has an energy gain of 10 dB relative to the ideal PM signal modem.

Single-channel modem A2-1

The new modem contains a modulator (Fig. 3.1) and a single-channel demodulator (Fig. 3.8). Its name will be: **modem A2-1**. The modulation algorithm for a quasi-deterministic signal (2.1) remains the same and is recorded in Table 4.1. At the same time, the above theoretical analysis of modem noise immunity when operating in a noisy channel remains unchanged for the new modem model. However, the new modem model has only one channel and one output. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator in Figure 3.7.

Table 4.3 shows that in the sinus channel of the demodulator, the logical "0" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. Table 4.4 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB, if the probability of errors $2 \cdot 10^{-45}$ is conventionally equated to zero. Theoretically, when these advantages of both channels are combined, they should get a new modem model with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, in practice this does not work out, which is confirmed by table 3.1 of truth. The probability of errors in modem A2 - 1 decreases on average by 20 times compared with the probability of errors in the cosine channel of modem A2.

Figure 4.3 shows the error probability of different new generation modems, where curve 1 is plotted for a known 4-QAM modulation and curve 3 for known QPSK modulation. Curves 2, 4 – 7 are plotted for the new SSK modulation. Curve 2 shows the error probability of modem A1 with a non-optimal modulation algorithm, and curve 4 - with the optimal modulation algorithm. Curve 5 refers to modem A2 (cosine channel), curve 7 - to modem A2 (sine channel). Curve 6 refers to an A2-1 dual-channel modem with channel bonding connected.

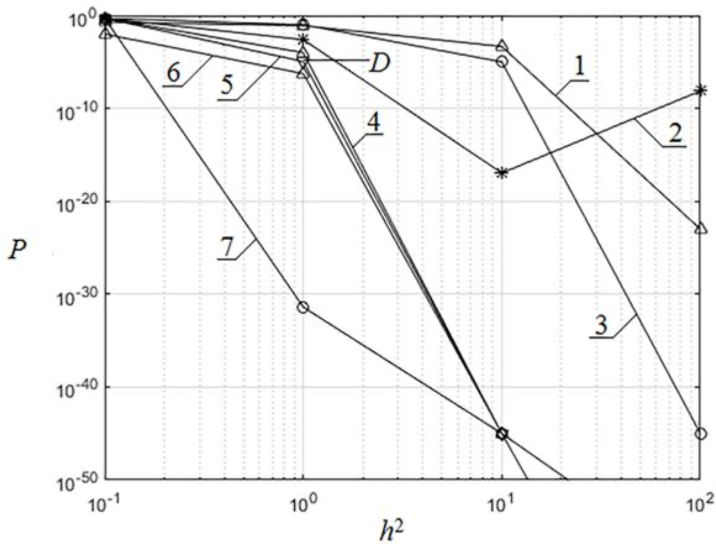


Figure 4.3. *The probability of errors of different modems of the new generation*

Modem A2–1 is superior in noise immunity to the cosine channel of modem A2 and modem A1. It has a potential noise immunity in the range of 30 dB and in this indicator exceeds, at least twenty orders of magnitude, modems known from domestic and foreign literature. The A2-1 modem with such characteristics has no analogues and competitors all over the world.

To test the theory, statistical modeling was first carried out, and then, for re-verification, simulation modeling of the A2 modem and the A2–1 modem, its results were published [24,30]. The simulation confirmed the results of the theoretical analysis of the noise immunity of the demodulator. There is marked point D in Figure 4.3 where the results of calculations and simulations coincided.

4.1.2. Noise immunity of A1 modem when receiving an additive mixture of noise and a centered signal with the distribution of instantaneous values according to the arcsine law

The modem contains a modulator, the quasi-deterministic signal at the output of which is shown in Figure 3.2e and coincides in shape with the classical amplitude keying, and the demodulator is single-channel (Figure 3.9). Its name will be: **modem A1**. The modulation algorithm for a quasi-deterministic signal (2.1) is written in Table 4.7.

Table 4.7.

Suboptimal signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "1"	1,125	0
logical "0"	0	0

The methodology and results of the studies were published in [25]. Let us turn to the analysis of the noise immunity of the demodulator, under the action of an additive mixture (4.1) of a centered quasi-deterministic signal (2.1) and "white" noise at its input

$$z(t)=u(t)+n(t),$$

where $n(t)$ – "white" noise, $u(t)$ – a signal with $a = U_0$.

Using expressions (3.3, 3.4) and the data in Table 4.7, using formulas (3.13), we calculate the threshold in the cosine channel of the demodulator. As a result, with the value $V_m = 1$, we get

$$П_2 = J_0(U_0,\text{t}) = J_0(0) = 1.$$

Further, with the value V_m=1 and $s(t)$ =1 let's define for the additive mixture (4.1)

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W_1(z)dz=J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)=$$

$$=J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right), \qquad (4.5a)$$

where $W_1(z)$ – probability density of instantaneous values of the additive mixture (4.1); $h=\sigma_c/\sigma_ш$ – signal-to-noise ratio; $\sigma_c^2=U_0^2/2$ – is the dispersion of the quasi-deterministic signal. If the signal *s(t)* =0, then expression (4.5a) takes a different form

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W_1(z)dz=J_0(0)\exp\left(-\frac{\sigma_ш^2}{2}\right)=\exp\left(-\frac{\sigma_c^2}{2h^2}\right). \qquad (4.5б)$$

The results (4.5a), (4.5b) require a quantitative analysis. Tables 4.8,4.9 present calculation data at $П_2$=1; $К_2$=0,55; $\sigma_c^2=1{,}125$, written in a line with the name evaluation.

Table 4.8.

Probability of errors at logical "1"

Threshold $П_{2k}$	$1\cdot 0{,}55=0{,}55$					
Evaluation $\widehat{A}(1,t)$	0	0	0,004	0.57	0,95	1
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$5\cdot10^{-3}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$

When analyzing the data in Table 4.8, we always compare the values of the ch.f. additive mixture with the threshold recorded in the first row of the table. At the same time, we see that the data in Table 4.8 exceed the threshold, starting from the signal-to-noise ratio of 1 to 100, i.e. in the range of 20 dB. This means that there will either be no errors here when accepting a logical “1”, or they will be minimal. Let's recall that the inverter is turned on at the demodulator output, therefore, the recipient of information has a logical "0".

Table 4.9.

Probability of errors at logical "0"

Threshold $П_{2k}$	$1\cdot 0{,}55=0{,}55$					
Evaluation $\widehat{A}(1,t)$	0	0	0,002	0,29	0,49	0,51
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	$2\cdot10^{-45}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$	$2{,}2\cdot10^{-17}$	$1{,}5\cdot10^{-8}$

The data in Table 4.9 came out below the set threshold. Therefore, they correspond to the ideal case. This allows us to say that accepting of the logical "0" in the demodulator occurs without errors, i.e. with maximum noise immunity in the range of signal-to-noise power ratios from 10^{-3} to 10^{2} or in the range of 50 dB. And, as a result of this, simple control commands (“turn on” or “turn off”, “open” or “close”, “raise” or “lower” and others) will be accepted by the modem with a reliability equal to one. If there is an inverter in the demodulator, the accepting of information receives the message in the form of a logical "1".

Let's move on from qualitative data analysis to a quantitative assessment of modem noise immunity. In tables 4.8,4.9, the following designations are accepted: P_0 - the probability of errors when accepting a logical "0"; P_1 - the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ total probability of device errors.

Quantitative assessment of noise immunity of modem A1

Again, we note that in expression (3.11), an ideal adder is used instead of the expectation operator. And, as a result of this, we obtain an estimate of the real part of the ch.f., which is recorded in tables 4.8,4.9. Repeating verbatim the reasoning stated above in section 4.1.1, we obtain, in relation to the data of tables 4.8, 4.9, the quantitative values of the error probability of the new modem model. The probability of demodulator errors depending on the signal-to-noise ratio with the help of graphs is shown in Figure 4.4, where curves 1 (at K_2=0.55), 2 (at K_2=0.53) characterize SSK (statistical shift keying) according to the data obtained here, curve 3 is amplitude keying according to the data of [15, p.478], curve 4 is the ideal PM according to the data of [15, p.473]. The variable coefficient K_2, significantly affects the noise immunity of the A1 modem, since up to the value $h^2 = 10$ inclusive, curve 1 looks better than curve 4, which characterizes the noise immunity of an ideal PM. The difference between them reaches ten orders of magnitude, and the gain in noise immunity belongs to amplitude manipulation. Here, the SSK competes as a leader, outperforming phase keying even in noise immunity. To detail the error probability, its main values are recorded in Table 4.10. For comparison, in the same place from [15, p.478], the probability of errors of ideal amplitude shift keying (*AM*), calculated in a noisy channel, is given. When the value of the coefficient K_2 =0.55, modem A1 is superior in noise immunity to the known device using the ideal *AM* in the range of signal-to-noise power ratios of 20 dB.

Table 4.10.

Probability of errors of different modems

New modem error probability (curve 1)	0,5	$5\cdot10^{-1}$	$2,5\cdot10^{-3}$	$1,1\cdot10^{-17}$	$7,5\cdot10^{-9}$
Probability of errors *AM*	0,82	0,62	$4,8\cdot10^{-1}$	$6\cdot10^{-2}$	$3\cdot10^{-7}$
Signal-to-noise ratio	0,1	0,5	1,0	10	100

Comparison of the noise immunity of the A1 modem with the noise immunity of the known device, in which the ideal *AM* is used, shows its superiority by at least two orders of magnitude or more, up to fifteen orders of magnitude.

Thus, new knowledge makes it possible to improve, at least two orders of magnitude, the noise immunity of modems with amplitude shift keying, which have been operating in digital communication systems over the past decades. For this, not much is required at all - to build a demodulator patented in Russia and conduct its full-scale tests.

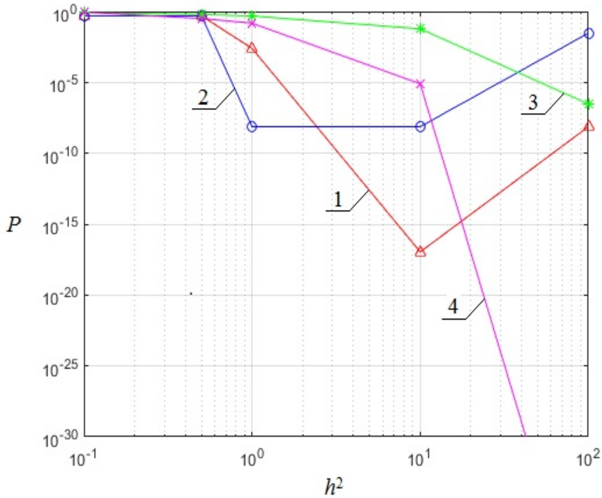


Figure 4.4. *Error probability of a single-channel modem A1*

First, statistical modeling was carried out, and then, for re-verification, simulation modeling of the A1 modem was carried out, its results were published [24, 30]. The simulation confirmed the results of the theoretical analysis of the noise immunity of the demodulator.

It seems that new knowledge allows, in our opinion, to make a big leap towards improving the noise immunity of old-generation modems operating with amplitude-keyed signals. Our conclusion can be treated differently. However, it cannot be left unattended, because in the era of the digital economy, a new generation modem has a great future.

Comparison of Figures 4.3, 4.4 shows that a single-channel A1 modem with a non-optimal modulation algorithm (Table 4.7, curve 1 in Fig. 4.4 or curve 2 in Fig. 4.3) is inferior in noise immunity to the same A1 modem with an optimal modulation algorithm (Table 4.11, curve 4 in Figure 4.3). The A1 modem with the optimal modulation algorithm is 10 dB more energy efficient than the known QPSK modulation.

Table 4.11.

Optimal signal modulation algorithm with V_m=2

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "1"	0,72	0
logical "0"	0	0

The search for optimal signal modulation algorithms is aimed at constructing such a special function $J_0(V_m U_0 s(t))$ in formulas (4.5a, 4.5b), the values of which at *s(t)* = 1 and *s(t)* = 0 differ from each other by several orders of magnitude.

4.2. Noise immunity of modem B when receiving an additive mixture of noise and signal with the distribution of instantaneous values according to the Veshkurtsev law

The statistical law of Veshkurtsev [12] has quantitative parameters that can be changed abruptly, i.e. modulate the signal with the distribution of instantaneous values according to this law. In total, we have considered 12 methods of modulating the characteristic function (ch. f.) of a quasi-deterministic signal with the distribution of its instantaneous values according to the Veshkurtsev law. The variables in this law were the variance and the expectation of the random amplitude of the signal, which, in turn, was distributed according to the Gauss law. The result is 6 direct signal modulation methods and 6 inverse modulation methods. The 10 signal modulation methods in the demodulator require sine and cosine channels to transform its instantaneous values in order to recover the transmitted information. Only 2 modulation methods require one cosine channel in the demodulator to recover the telegraph signal.

4.2.1. Noise immunity of the B1 modem when receiving an additive mixture of noise and signal with the distribution of instantaneous values according to the centered Veshkurtsev law

The modem contains a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.9). Its name will be: **modem B1**. The modulation algorithm for a quasi-deterministic signal (2.9) is written in Table 4.12.

Table 4.12.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "1"	1,0	0
logical "0"	0,0009	0

The research methodology and results are published in [10,12,31]. Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a quasi-deterministic signal (2.9) and "white" noise acts at its input

$$z(t)=u(t)+n(t), \tag{4.6}$$

where $n(t)$ – "white" noise, $u(t)$ – signal (2.9).

Using expression (2.12) and the data in Table 4.11, using formula (3.13), we calculate the threshold in the demodulator. As a result, with the value $V_m = 1$, we get

$$П1=I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right)=0{,}7917.$$

Let us represent the functional transformation in the demodulator circuit with the dependence y =cosz at the value $V_m = 1$ and N>> 1. Let us calculate the expectation $m_1\{y\}$, since the ch.f. is the expectation of the cosine function for the real part and the sine function for the imaginary part. Let's recall that the imaginary part of the ch.f. is equal to zero. We get with the value $V_m = 1$

$$m_1\{y\}=\int_{-\infty}^{\infty}\cos(z)W(z)dz=I_0\left(\frac{1}{4}\sigma^2\right)\exp\left[-\left(\frac{\sigma^2+2\sigma_ш^2}{4}\right)\right], \tag{4.7}$$

where $W(z)$ - the probability density of the additive mixture (4.6); $\sigma_ш^2$ - dispersion of "white" noise. Dispersion of the modulated c.c.s. changes abruptly from $\sigma_0 2$ to σ_1^2, the values of which are recorded in Table 4.12. Then, when transmitting a logical "0", we get

$$m_1\{y\}_0=I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\left(\frac{\sigma_0^2+2\sigma_ш^2}{4}\right)\right], \tag{4.8}$$

and when transmitting a logical "1" will be

$$m_1\{y\}_1=I_0\left(\frac{\sigma_1^2}{4}\right)\exp\left[-\left(\frac{\sigma_1^2+2\sigma_ш^2}{4}\right)\right], \tag{4.9}$$

Having performed the following substitutions in expressions (4.8,4.9) $\sigma_ш^2=\sigma_0^2/h_0^2$, $\sigma_ш^2=\sigma_1^2/h_1^2$, we get

$$A(1,t)=I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h_0^2}{4h_0^2}\right)\right], \tag{4.10}$$

$$A(1,t)=I_0\left(\frac{\sigma_1^2}{4}\right)\exp\left[-\sigma_1^2\left(\frac{2+h_1^2}{4h_1^2}\right)\right], \tag{4.11}$$

where $h_0 = \sigma_0 / \sigma_ш$ - signal-to-noise ratio when receiving logical "0";

$h_1 = \sigma_1 / \sigma_ш$ - signal-to-noise ratio when receiving a logical "1".

The results (4.10), (4.11) require a quantitative analysis. Tables 4.13, 4.14 show the calculation data at K_1 =1,14, $П_1$ = 0,7917, recorded in the line with the name of the evaluation. In this case the following is taken into account. The modulation algorithm in Table 4.12 contains σ_0^2 and σ_1^2. This means that at a constant noise power in the case of transmission of a logical "0" and a logical "1", the ratio $h_0^2 > h_1^2$. For the given values of the dispersions, we will obtain a value $h_0^2 = 1111{,}11\ h_1^2$ and use it in the calculations. When the modem is operating in a noisy channel, it is impossible to provide a different signal-to-noise ratio at its input when receiving a logical "0" and a logical "1", because the noise power in the channel does not depend on the logical "0" and "1". Therefore, the final conclusions about the probability of modem errors in this case should be taken depending on the values of the ratio h_1^2.

Table 4.13.

Probability of errors at logical "1"

Threshold $П_{1c}$	0,7917·1,14 = 0,9					
Evaluation $\hat{A}(1,t)$	0,6376	0,9512	1	1	1	1
Relation h_1^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	$1{,}5\cdot10^{-12}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$

In Table 4.13, the evaluation values $\hat{A}(1,t)$ exceed the threshold at a signal-to-noise ratio from 0.01 to 100; here, in a noisy channel, there are no errors when accepting a logical "1" in the demodulator. When $h_1^2 < 0{,}01$ errors appear in the demodulator in the channel with noise when accepting a logical "1". Thus, the range of signal-to-noise ratios is only 40 dB, with the lower limit of the range being minus 20 dB.

Table 4.14.

Probability of errors at logical "0"

Threshold $П_{1c}$	0,7917·1,14 = 0,9					
Evaluation $\hat{A}(1,t)$	0,505	0,757	0,788	0,792	0,792	0,792
Relation h_0^2	1,11111	11,1111	111,111	1111,11	11111,1	111111
Probability of errors P_0	0	0	0	0	0	0

In Table 4.14, all evaluation values $\hat{A}(1,t)$ are less than the threshold for any signal-to-noise ratio. This means that in a channel with noise, the demodulator does not have errors when receiving a logical “0” in the range of signal-to-noise ratios of 50 dB.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the B1 modem. In tables 4.13,4.14, the following designations are accepted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - the total probability of device errors.

Quantitative assessment of modem noise immunity B1

In expression (3.11), an ideal adder is used instead of the expectation operator. And, as a result of this, we obtain an estimate of the real part of the ch.f., which is written in tables 4.13, 4.14. An estimate is a random variable that has its own properties and distribution law. Repeating verbatim the rationale and methodology for calculating errors in the demodulator, set out in Section 4.1.1, we get the data recorded in Table 4.15. For comparison, the error probability of ideal phase modulation (PM) is given in the same place from [15, p.473], calculated in a noisy channel.

Table 4.15.

Probability of errors of different modems

Probability of modem errors	**$7{,}5\cdot10^{-13}$**	**$1\cdot10^{-45}$**	**Less than $1\cdot10^{-45}$**	**Less than $1\cdot10^{-45}$**	**Less than $1\cdot10^{-45}$**
PM error probability	1,0	0,9	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$
Signal-to-noise ratio	0,01	0,1	1,0	10	100

The error probability of the demodulator depending on the signal-to-noise ratio with the help of graphs is shown in Figure 4.5, where curve 1 characterizes the error probability according to the data obtained here, curve 2 - the ideal PM according to the data of [15, c.473]. The variable coefficient K_1 significantly affects the probability of modem errors. Thanks to it, you can adjust the amount of demodulator errors.

Comparison of the noise immunity of a new modem model with the noise immunity of a well-known device in which ideal PM is used shows its superiority by at least ten orders and even up to thirty orders. These figures are simply fantastic. Perhaps they determine the potential noise immunity of statistical modulation for the near future, which must be achieved. Therefore, in a noisy channel, modem B1 has maximum noise immunity or, in other words, it works without errors when receiving a telegraph message in the range of signal-to-noise ratios from 0.01 to 100, i.e. in the range of 40 dB, starting from a value of minus 20 dB. This indicates that the expectation operator in the mathematical model of ch.f. reliably protects the signal from noise. Modems of the new generation can work without errors when the signal-to-noise ratio is much less than one. This modem model can be used in radio and wired, cable, fiber-optic communication channels. This modem has no analogues and competitors all over the world.

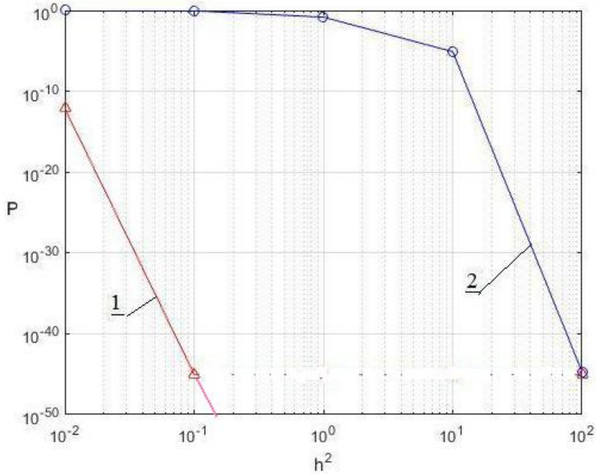


Figure 4.5. *Error probability of a single-channel modem B1*

4.2.2. Noise immunity of the B2 modem when accepting an additive mixture of noise and signal with the distribution of instantaneous values according to the non-centered Veshkurtsev law

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its name will be: **modem B2**. The modulation algorithm for a quasi-deterministic signal (2.9) is written in Table 4.16.

Table 4.16.
Signal modulation algorithm with V_m*=1*

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,01	0
logical "1"	0,01	0,6

The research methodology and results are published in [10,13,31]. Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a quasi-deterministic signal (2.9) and "white" noise acts at its input

$$z(t)=u(t)+n(t), \qquad (4.12)$$

where $n(t)$ – "white" noise, $u(t)$ – signal (2.9).

Using expressions (2.12,3.10) and the data in Table 4.16, using formulas (3.13), we calculate the thresholds in the demodulator. As a result, with the value $V_m = 1$, we get

$$П1= I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right)\sin(e_0) = 0{,}5646\ , \qquad П2= I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right) = 1.$$

Let us calculate the real and imaginary parts of the ch.f. additive mixture (4.12) and is comparable with the thresholds. Then, during the transmission *s(t)* = 0 and value V_m = 1 in the channels of the demodulator, the threshold devices will accept the values of the real and imaginary parts of the ch.f. additive mixture equal to

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right],\ B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z)dz = 0, \quad (4.13)$$

where $W(z)$ - the probability density of the additive mixture; $h = \sigma_0/\sigma_ш$ - signal-to-noise ratio; $\sigma_ш{}^2$ - dispersion of "white" noise. When transmitting *s(t)* = 1 and the value V_m = 1 in the channels of the demodulator, the threshold devices will accept the values of the real and imaginary parts of the ch.f. additive mixture, equal

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z+e_0)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\cos(e_0), \quad (4.14)$$

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z+e_0)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\sin(e_0). \quad (4.15)$$

Let $K_2 = 0{,}96; K_1 = 0{,}532; П_2 = 1; П_1 = 0{,}5646; \sigma_0^2 = 0{,}01; e_0 = 0{,}6$. The results of calculations by formulas (4.13,4.14,4.15) are summarized in tables 4.17, 4.18,4.19,4.20, recorded in the line with the name of the evaluation.

Table 4.17.

Probability of errors in the cosine channel of the modem at a logical "1"

Threshold $П_{2k}$	1· 0,96 = 0,96					
Evaluation $\hat{A}(1,t)$	0,0067	0,6065	0,9512	0,99	1	1
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	0,988	$2{,}1\cdot10^{-5}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$

Analysis of the data in tables 4.17–4.20 shows that the logical "1" in the cosine channel and the logical "0" in the sine channel are determined correctly, i.e. without errors, at any signal-to-noise ratio in the range from 10^{-3} to 10^2 or 50 dB in power. Logical "0" in the cosine channel is determined without errors when the signal-to-noise ratio is from 0.1 to 100, i.e. in the range of 30 dB in power.

Table 4.18.

Probability of errors in the cosine channel of the modem with a logical "0"

Threshold Π_{2k}	1· 0,96 = 0,96					
Evaluation $\hat{A}(1,t)$	0,0056	0,5005	0,785	0,817	0,8253	0,8253
Relation h^2	0,001	0,01	0,1	1.0	10	100
Probability of errors P_0	0	0	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$	$2{\cdot}10^{-45}$

Table 4.19.

Probability of errors in the sinus channel of the modem at a logical "1"

Threshold Π_{1c}	0,532·0,5646 = 0,3					
Evaluation $\hat{B}(1,t)$	0,0038	0,3424	0,537	0,559	0,5646	0,5646
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	$5{,}8{\cdot}10^{-2}$	$2{,}8{\cdot}10^{-26}$	$4{,}3{\cdot}10^{-31}$	$1{,}5{\cdot}10^{-32}$	$1{,}5{\cdot}10^{-32}$

Table 4.20.

Probability of errors in the sinus channel of the modem at a logical "0"

Threshold Π_{1c}	0,532·0,5646 = 0,3					
Evaluation $\hat{B}(1,t)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	$3{,}8{\cdot}10^{-41}$	$3{,}8{\cdot}10^{-41}$	$3{,}8{\cdot}10^{-41}$	$3{,}8{\cdot}10^{-41}$	$3{,}8{\cdot}10^{-41}$	$3{,}8{\cdot}10^{-41}$

At the same time, in the sinus channel, the logical "1" is determined without errors when the signal-to-noise ratio is from 0.01 to 100, i.e. in the range of 40 dB on power. In this modem, the sinus channel prevails, since it has a maximum noise immunity in the range of signal-to-noise ratio of 40 dB when operating in a communication channel with white noise interference, and the lower limit of the range is minus 20 dB.

From a qualitative analysis of the data, let's move on to a quantitative assessment of the noise immunity of the B2 modem. In tables 4.17 - 4.20, the following designations are adopted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ – total probability of device errors.

Quantitative assessment of modem noise immunity B2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we get evaluations of the real and imaginary parts of the ch.f., which are recorded in tables 4.17 - 4.20. Both evaluations are random variables with their own properties and distribution laws. Repeating verbatim the rationale and methodology for calculating errors in the demodulator channels, set out in Section 4.1.1, we obtain the data recorded in Table 4.21. For comparison, the error probability of ideal phase modulation (PM) in the same place from [15, p.473] is given, calculated in a noisy channel.

Table 4.21.

Probability of errors of different modems

Total sinus channel error probability	**$2{,}9\cdot10^{-2}$**	**$1{,}4\cdot10^{-26}$**	**$2{,}1\cdot10^{-31}$**	**$7{,}5\cdot10^{-33}$**	**$7{,}5\cdot10^{-33}$**
Total cosine channel error probability	0,5	0,5	$1\cdot10^{-5}$	$7{,}5\cdot10^{-9}$	$7{,}5\cdot10^{-9}$
PM error probability	1,0	0,9	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$
Signal-to-noise ratio	0,01	0,1	1,0	10	100

The dependence of the B2 modem error probability on the signal-to-noise ratio is shown in Figure 4.6, where curves 1,2 refer to the cosine channel of the demodulator; curves 3,4 - to the sinus channel of the demodulator; curve 5 - to the ideal PM. The sine and cosine channels have different noise immunity, and in each channel it strongly depends on the value of the thresholds (3.13). With the help of variable coefficients K_1, K_2, the modem can be configured. An example of this is shown in Figure 4.6, where curve 2 is obtained with a coefficient K_1 = 0.52; curve 4 - with coefficient K_1=0.532; curve 1 - with coefficient K_2 = 0.98; curve 3 - with coefficient K_2 = 0.96. Curves 1, 2 have the expectation of the signal e_0 =0.4, and curves 3,4 have the value e_0 =0.6, which is recorded in Table 4.16. This means that changing the value of e_0 also affects the noise immunity of modem B2. Therefore, it is necessary to create the foundations of the theory of statistical signal modulation algorithms with the Veshkurtsev distribution law in order to obtain good noise immunity of the B2 modem. Unfortunately, this has not been done yet, and the signal modulation algorithm is written approximately. Nevertheless, the probability of errors in the sinus channel of the modem turned out to be tremendously low and lies at the level of $7.5\cdot10^{-33}$ when receiving weak signals, when the signal-to-noise power ratio does not exceed 17 dB, and the lower limit of the ratio is minus 10 dB. According to this indicator, the B2 modem is thirty orders of magnitude superior to a similar device with ideal phase modulation (curve 5). On the other hand, the cosine channel of modem B2 has an error probability of $7.5\cdot10^{-9}$ and competes very little with the well-known device using PM. Therefore, it has no prospects for existence in the future.

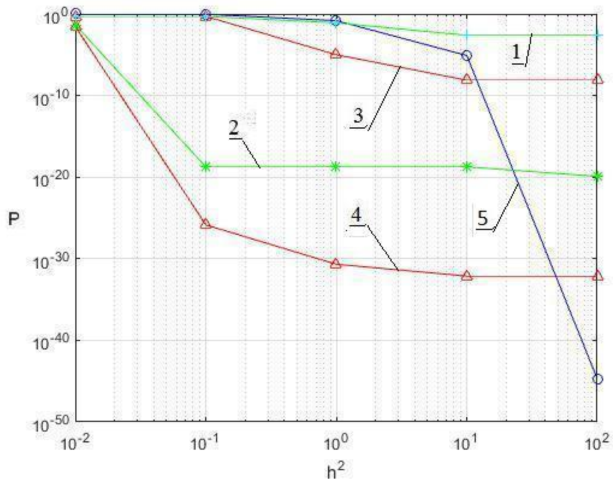


Figure 4.6. *Error probability of a dual-channel modem B2*

Single-channel modem B2-1

The new modem contains a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). Its name will be: **modem B2-1.** The modulation algorithm for a quasi-deterministic signal (2.9) is written in Table 4.16. At the same time, the above theoretical analysis of modem noise immunity when operating in a noisy channel remains unchanged for the new modem model. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of the execution of the state table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator shown in Figure 3.7.

Table 4.20 shows that in the sinus channel of the demodulator, the logical "0" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB, if the probability of errors $3.8 \cdot 10^{-41}$ is conventionally equated to zero. Table 4.18 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB, if the probability of errors $2 \cdot 10^{-45}$ is conventionally equated to zero. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, this theoretical result cannot be realized in practice. The demodulator in Figure 3.8 improves the noise immunity of the B2 modem's cosine channel by only a factor of 20 on average.

The error probability of different modems of the new generation is shown in Figure 4.7, where curve 3 refers to modem B2 - 1, curve 1 - to modem B1. It also shows the error probability of the sine channel (curve 2) and the cosine channel (curve 4) of the modem B2, as well as the known device (curve 5) for accepting signals with phase modulation.

The B2-1 modem is superior in noise immunity to the cosine channel of the B2 modem, but inferior to the sine channel of the B2 modem, the B1 modem and the device using phase modulation. Such a modem has no prospects for the future.

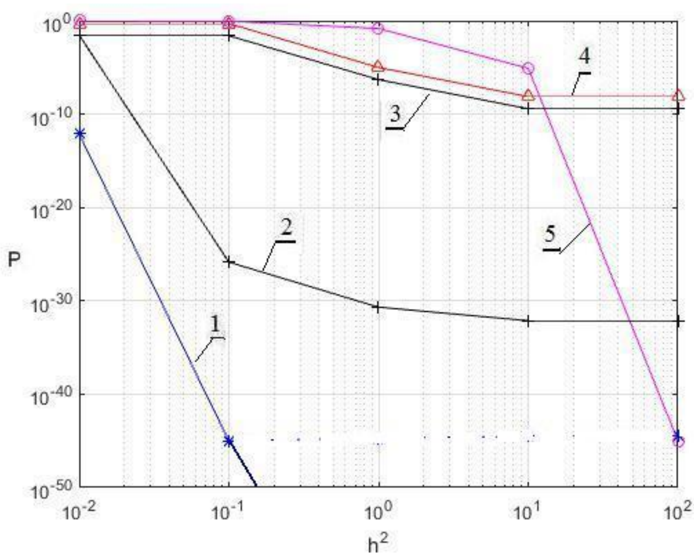


Figure 4.7. *The probability of errors of different modems of the new generation*

4.3. Noise immunity of the modem K2 when accepting an additive mixture of noise and signal with the distribution of instantaneous values according to the cosine law

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its name will be: **modem K2**. The modulation algorithm for a quasi-deterministic signal (2.19) is written in Table 4.22.

Table 4.22.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,4674	0
logical "1"	0,4674	0,8

The modem research technique was developed in [14]. Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a quasi-deterministic signal (2.19) and "white" noise acts at its input

$$z(t)=u(t)+n(t), \quad (4.16)$$

where *n(t)* – "white" noise, *u(t)* – a signal with $a=U_0$.
Using expressions (2.26,2.27) and the data in Table 4.22, using formulas (3.13), we calculate the thresholds in the demodulator. As a result, with the value $V_m = 1$, we get

$$П_1 = \frac{\pi}{4}\sin(e_0) = 0{,}5634\,,\ П_2 = \pi/4 = 0{,}7854.$$

For the value V_m=1, we define for the additive mixture (4.16) the real part of the ch.f.

$$A\,(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z)dz = \frac{\pi}{4}\exp\left(-\frac{\sigma_c^2}{2h^2}\right), \quad (4.17)$$

where $h = \sigma_c/\sigma_ш$ - signal-to-noise ratio. When *s(t)*=0, similarly to (4.17) we calculate for the value V_m=1 for the additive mixture (4.16) the imaginary part of the ch.f.

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z)dz = 0, \quad (4.18)$$

The results (4.17), (4.18) require a quantitative analysis. Tables 4.23, 4.24 present the results of calculations at $П_1 = 0{,}5634$, $П_2 = 0{,}7854$, $K_1 = 0{,}53$, $K_2 = 0{,}764$, written in a line with the name of the evaluation.

Table 4.23.

Probability of errors in the cosine channel of the modem at a logical "1"

Threshold $П_{2k}$	0,7854· 0,764 = 0,6					
Evaluation $\hat{A}(1,t)$	0	0	0,08	0,624	0,77	0,79
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$6{,}9\cdot10^{-4}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$

Table 4.24.

Probability of errors in the sinus channel of the modem at a logical "0"

Threshold Π_{1c}	0,5634·0,53 = 0,3					
Evaluation $\hat{B}(1,t)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	0	0	0

Analysis of the data in Table 4.23 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors in the range of signal-to-noise power ratios from 1 to 100 or from 0 dB to 20 dB. Table 4.24 presents the ideal results, as logical "0" in the sinus channel of the demodulator is determined without errors, i.e. with ultimate noise immunity, at any signal-to-noise power ratio in the range of 50 dB. This allows us to say that simple control commands such as turn on-off, open-close and others will be accepted with a reliability equal to one, in any operating conditions of the K2 modem.

Suppose the additive mixture (4.16) at the demodulator input contain non-centered quasi-deterministic signal, this corresponds to the condition *s(t)*=1. Similarly to (4.17), at the value V_m=1 we define

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z)dz=\frac{\pi}{4}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(e_0) \tag{4.19}$$

or similarly (4.18) with the value V_m=1 we calculate

$$B(1,t)=\int_{-\infty}^{\infty}\sin(z)W(z)dz=\frac{\pi}{4}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin(e_0 \tag{4.20}$$

The results (4.19), (4.20) require a quantitative analysis. Tables 4.25, 4.26 show calculation data at $\Pi_1=0{,}5634$, $\Pi_2=0{,}7854$, $K_1=0{,}53$, $K_2=0{,}764$, written in a string with the name of the evaluation.

Table 4.25.

Probability of errors in the sinus channel of the modem at a logical "1"

Threshold Π_{1c}	0,5634· 0,53 = 0,3					
Evaluation $\hat{B}(1,t)$	0	0	0,05	0,44	0,55	0,56
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$1{,}1\cdot10^{-12}$	$8\cdot10^{-32}$	$8{,}6\cdot10^{-35}$

In case of the selected threshold values according to the data of tables 4.25, 4.26, the distinction of logical "1" from zero in the sinus channel of the demodulator occurs without errors in the range of signal-to-noise ratios from 1 to 100, i.e. in the range equal to 20 dB. In this case, in the cosine channel of the demodulator the maximum noise immunity is maintained at a signal-to-noise power ratio of 0.001 and higher up to 100, i.e. in the 50 dB range, for which the lower limit is minus 30 dB. Therefore, simple control commands such as turn on-off, close-open and others will be accepted by the cosine channel with a reliability equal to one, under any operating conditions of the K2 modem.

Table 4.26.

Probability of errors in the cosine channel of the modem at a logical "0"

Threshold Π_{2k}	0,7854· 0,764 = 0,6					
Evaluation $\hat{A}(1,t)$	0	0	0,05	0,43	0,54	0,55
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	$2·10^{-45}$	$2,2·10^{-17}$	$1,5·10^{-12}$

As a result of the analysis of the noise immunity of the K2 modem, we can say that in the presence of "white" noise in the data transmission channel, the noise immunity according to Kotelnikov of the proposed modem is limiting. With accurate synchronization of the operation of both channels of the K2 modem, there are no errors when receiving a telegraph signal in the range of signal-to-noise ratios of 20 dB or more, and the lower limit of the range is 0 dB.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the K2 modem. In tables 4.23-4.26, the following designations are accepted: P_0 – the probability of errors when receiving a logical "0"; P_1 – the probability of errors when receiving a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - the total probability of device errors.

Quantitative assessment of the noise immunity of the K2 modem

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 4.23 - 4.26. Both estimates are random variables with their own properties and distribution laws. Let's recall that evaluations for the real and imaginary parts of the ch.f. are efficient, consistent, and unbiased. This is shown in earlier works, for example [2], in which the effectiveness of estimates is characterized by their variances.

Repeating verbatim the reasoning stated above in section 4.1.1, we get, in relation to the data of tables 4.23 - 4.26, the quantitative values of the error probability of the new modem model. The probability of demodulator errors depending on the signal-to-noise ratio with the help of graphs is presented in Figure 4.8. Curve 1 characterizes the error probability of the sine channel, curve 2 characterizes the cosine channel of the K2 modem, curve 3 characterizes the device in which PM is applied.

The total error probability of the sine and cosine channels of the K2 modem is shown in Table 4.27. For comparison, in the same place from [15, p. 473] there is presented the error probability of ideal phase modulation (PM) calculated in a noisy channel.

Comparison of the noise immunity of a new modem with the noise immunity of a well-known device in which an ideal PM, shows its superiority by thirteen orders and up to thirty orders. Modem K2 in the cosine channel has a reference point with an error probability of a value $P = 3{,}5 \cdot 10^{-4}$., different from zero, i.e. from the maximum noise immunity of the device. Its occurrence may be associated with a random, without any justification, choice for modulating the quantitative parameters of the distribution law of a quasi-deterministic signal. It is likely that the optimization of these parameters using the newly constructed theory of statistical modulation will eliminate the modem reference point.

Table 4.27.

Probability of errors of different modems

Total sinus channel error probability	**0,5**	**$9 \cdot 10^{-3}$**	**$5{,}5 \cdot 10^{-13}$**	**$4 \cdot 10^{-32}$**	**$4{,}3 \cdot 10^{-35}$**
Total cosine channel error probability	0,5	0,5	$3{,}5 \cdot 10^{-4}$	$1{,}1 \cdot 10^{-17}$	$7{,}5 \cdot 10^{-13}$
PM error probability	0,9	$3{,}2 \cdot 10^{-1}$	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$
Signal-to-noise ratio	0,1	0,5	1,0	10	100

There are no fixed points in the sinus channel of the modem. Therefore, even with such data as they are recorded in table 4.27, one can hope for a good future for the K2 modem.

Comparison of the noise immunity of the K2 modem with the previously considered modems A2, A2-1 shows that it is lower, since the A2-1 modem under equal conditions has a maximum noise immunity in the range of signal-to-noise ratios of 20 dB.

The difference between the two compared modems lies only in the models of quasi-deterministic signals used in them. Apparently, a quasi-deterministic signal with an arcsine distribution law has an entropy less than an oscillation (2.19) with a cosine distribution law or a random process (2.9) with the Veshkurtsev distribution law, which has no equal among the distribution laws of signals considered in Chapter 2.

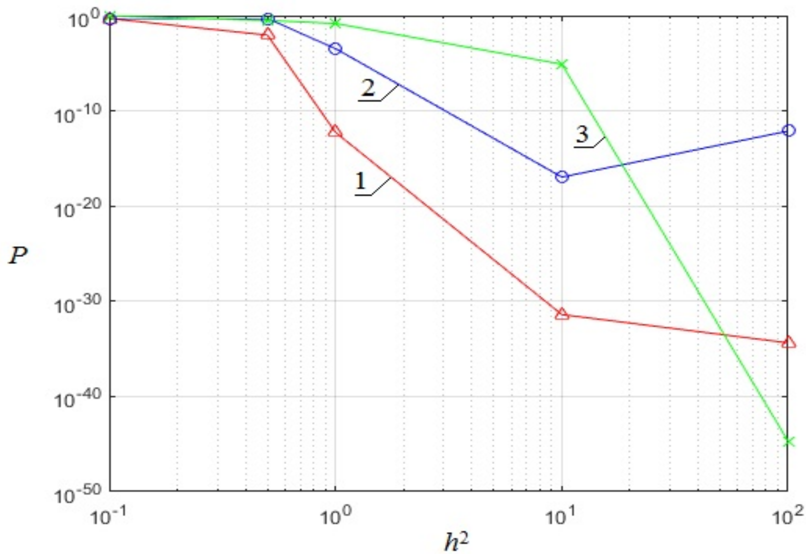


Figure 4.8. *Probability of modem K2 errors*

Single-channel modem K2 -1

Suppose the new modem contain a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). The modulation algorithm for a quasi-deterministic signal (2.19) is written in Table 4.22. At the same time, the above analysis of modem noise immunity when operating in a noisy channel remains unchanged for the new modem model. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of fulfilling the positions of the truth table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator shown in Figure 3.7.

Table 4.24 shows that in the sinus channel of the demodulator, the logical "0" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. Table 4.26 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, this theoretical result cannot be realized. The demodulator in Figure 3.8 improves the noise immunity of the cosine channel of the K2 modem on average only 20 times.

The error probability of the K2 modem and the K2-1 modem is shown in Figure 4.9, where curve 1 refers to the sinus channel of the K2 modem; curve

3 - to the cosine channel of modem K2; curve 2 - to modem K2-1; curve 4 - to the device in which phase modulation is applied.

An analysis of the graphs in Figure 4.9 shows that the K2-1 modem works well with weak signals and has a minimum error probability of $1 \cdot 10^{-19}$ at a signal-to-noise ratio of 10 dB, and then the error probability begins to increase to a value of $1 \cdot 10^{-14}$ if the signal is growing. Modem K2–1 is inferior in noise immunity to the sinus channel of modem modem K2 up to the signal-to-noise ratio of 20 dB. However, both modems are superior in noise immunity to the device for receiving signals with ideal PM (curve 4) in the range of signal-to-noise power ratios $0{,}1 \le h^2 \le 20$. Let's recall that the K2 modem and the modem K2–1 modem use statistical modulation of a quasi-deterministic signal. Therefore, the K2 modem and the K2-1 modem have no analogues and competitors all over the world.

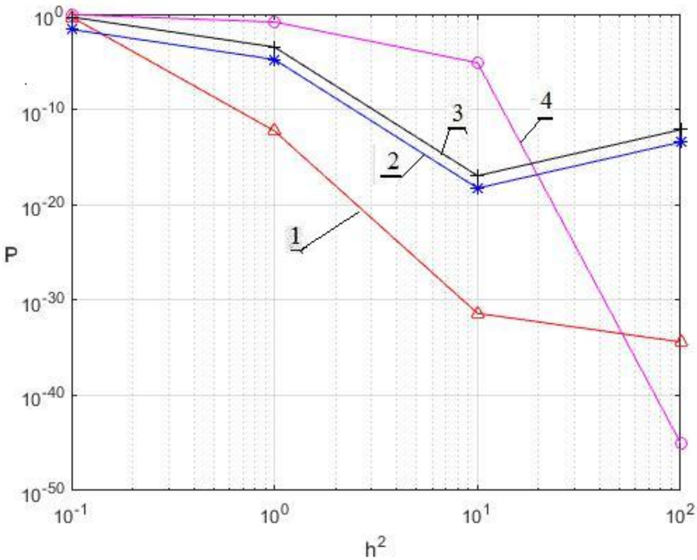


Figure 4.9. *The probability of errors of different modems of the new generation*

4.4. Modem noise immunity when accepting an additive mixture of noise and signal with the distribution of instantaneous values according to Tikhonov law

Tikhonov law (2.34) contains the parameter *D*, which is included in all the probabilistic characteristics of the signal. Parameter value $0 \le D \le \infty$. Let's look at a few examples.

4.4.1. Noise immunity of the modem T2 when accepting an additive mixture of noise and signal with the distribution of instantaneous values according to Tikhonov law with the parameter *D* = 1

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its name will be: **modem T2**. The modulation algorithm for a quasi-deterministic signal (2.33) is recorded in Table 4.28. The method and results of modem studies are published in [32].

Table 4.28.
Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	1,604	0
logical "1"	1,604	0,8

Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a quasi-deterministic signal (2.33) and "white" noise acts at its input

$$z(t)=u(t)+n(t), \tag{4.21}$$

where *n(t)* – "white" noise, *u(t)* – signal (2.33).

With the help of expressions (2.26,2.27) and the data in Table. 4.27 using formulas (3.13) we calculate the thresholds in the demodulator. As a result, at a value of V_m = 1 and D = 1 we get

$$\Pi_1 = \frac{I_1(D)}{I_0(D)}\sin(e_0) = 0{,}32\,, \qquad \Pi_2 = \frac{I_1(D)}{I_0(D)} = 0{,}4464.$$

At the value V_m=1 we define for the additive mixture (4.21) the real part of the ch.f.

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z)dz = \frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right), \tag{4.22}$$

where $h = \sigma_c/\sigma_ш$ - the signal-to-noise ratio. When *s(t)*=0, similarly to (4.22) we calculate at the value V_m=1 for the additive mixture (4.21) the imaginary part of the ch.f.

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z)dz = 0, \tag{4.23}$$

The results (4.22), (4.23) require a quantitative analysis. Tables 4.29, 4.30 present the results of calculations at $\Pi_1 = 0{,}32$, $\Pi_2 = 0{,}4464$, $K_1 = 0{,}375$, $K_2 = 0{,}448$, written in a line with the name of the evaluation.

Table 4.29.

Probability of errors in the cosine channel of the modem at a logical "1"

Threshold Π_{2k}	0,4464· 0,448 = 0,2					
Evaluation $\hat{A}(1,t)$	0	0	0	0,201	0,412	0,446
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$8{,}9\cdot10^{-1}$	$2\cdot10^{-45}$	$2\cdot10^{-45}$

Table 4.30.

The probability of errors in the sinus channel of modem at a logical "0"

Threshold Π_{1c}	0,32·0,375 = 0,12					
Evaluation $\hat{B}(1,t)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-8}$

Analysis of the data in Table 4.29 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors in the range of signal-to-noise power ratios from 1 to 100 or from 0 dB to 20 dB. Table 4.30 presents the ideal results, as logical "0" in the sinus channel of the demodulator is determined without errors, i.e. with ultimate noise immunity, at any signal-to-noise power ratio. This allows us to say that simple control commands such as turn on-off, open-close and others will be accepted with a reliability equal to one, in any operating conditions of the sinus channel of the modem T2.

Suppose the additive mixture (4.21) contain a non-centered quasi-deterministic signal at the demodulator input, this corresponds to the condition *s(t)*=1. Similarly to (4.22) at the value V_m=1 we define

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z-e_0)dz=\frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(e_0) \tag{4.24}$$

or similarly (4.23) with the value V_m=1 we calculate

$$B(1,t)=\int_{-\infty}^{\infty}\sin(z)W(z-e_0)dz=\frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin(e_0)\,. \tag{4.25}$$

The results (4.24), (4.25) require a quantitative analysis. Tables 4.31, 4.32 show the calculation data at $\Pi_1=0{,}32$, $\Pi_2=0{,}4464$, $K_1=0{,}375$, $K_2=0{,}448$, written in the line with the name of the evaluation.

Table 4.31.

Probability of errors in the sinus channel of the modem at a logical "1"

Threshold $П_{1c}$	0,32·0,375 = 0,12					
Evaluation $\widehat{B}(1,t)$	0	0	0	0,144	0,296	0,32
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$4\cdot10^{-1}$	$1\cdot10^{-16}$	$1\cdot10^{-21}$

In case of the selected values of the thresholds according to the data of tables 4.31, 4.32, the discrimination of the logical "1" from zero in the sinus channel of the modem T2 occurs without errors in the range of signal-to-noise ratios from 1 to 100, i.e. in the range equal to 20 dB. In this case, in the cosine channel of the demodulator, the limiting noise immunity is maintained at a signal-to-noise power ratio from 0.001 and above to 1.0, i.e. from minus 30 dB to 0 dB. Therefore, simple control commands such as turn on-off, close-open and others will be received by the cosine channel of the modem T2 with a reliability equal to one in the range of signal-to-noise ratios of 30 dB. If the signal-to-noise ratio is greater than ten, there will be continuous errors in the cosine channel of the demodulator, i.e. the operation of this channel of the modem T2 in the channel with noise becomes impossible.

Table 4.32.

Probability of errors in the cosine channel of the modem at logic "0"

Threshold $П_{2k}$	0,4464·0,448 = 0,2					
Evaluation $\widehat{A}(1,t)$	0	0	0	0,14	0,28	0,31
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	$2{,}2\cdot10^{-17}$	0,5	0,5

As a result of the analysis of the noise immunity of the modem, we can say that in the presence of "white" noise in the data transmission channel, the noise immunity according to Kotelnikov of the modem T2 turned out to be different in both channels. With accurate synchronization of the operation of the sinus channel of the modem T2, there are no errors when receiving a telegraph signal in the range of signal-to-noise ratios of 20 dB or more, and the lower limit of the range is 0 dB. But the cosine channel of the modem T2 has an error probability of 0.5 and cannot work here.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the modem T2. In tables 4.29-4.32, the following designations are adopted: P_0 – the probability of errors when accepting a logical "0";

P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - the total probability of device errors.

Quantitative assessment of the noise immunity of the modem T2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we get estimates of the real and imaginary parts of the ch.f., recorded in tables 4.29 - 4.32. Both evaluations are random variables with their own properties and distribution laws. Let's recall that evaluations for the real and imaginary parts of the ch.f. are efficient, consistent, and unbiased. This is shown in earlier works, for example [2], in which the effectiveness of estimates is characterized by their variances. Repeating verbatim the reasoning stated above in section 4.1.1, we get, in relation to the data of tables 4.29 - 4.32, the quantitative values of the error probability of the new modem model.

The total error probability of the sine and cosine channels of the modem T2 is shown in Table 4.33. For comparison, in the same place from [15, p. 473] there is presented the error probability of ideal phase modulation (PM) calculated in a noisy channel. In addition, for clarity, the probability of modem errors using graphs is shown in Figure 4.10.

Table 4.33.

Probability of errors of different modems

Total sinus channel error probability	0,5	0,5	$2 \cdot 10^{-1}$	$7{,}5 \cdot 10^{-9}$	$7{,}5 \cdot 10^{-9}$
Total cosine channel error probability	0,5	0,5	0,45	0,5	0,5
PM error probability	0,5	0,5	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$
Signal-to-noise ratio	0,01	0,1	1,0	10	100

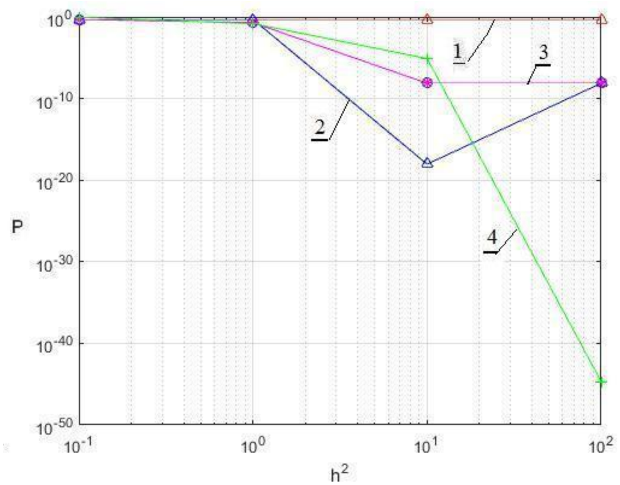


Figure 4.10. *Probability of modem T2p errors*

In Figure 4.10, curve 1 characterizes the cosine channel of the T2 modem at modem K_2 = 0.448, curve 2 - the cosine channel of the T2 modem at modem K_2 = 0.78 according to [30], curve 3 - the sine channel of the T2 modem at K_1 = 0.375, curve 4 - device for accepting PM signals. Comparison of the noise immunity of the modem with the noise immunity of the known device, in which the ideal PM is used, shows the superiority of the sinus channel characteristics in comparison with the prototype within the values $1 \le h^2 \le 10$, and then it disappears. The gain in noise immunity is only 0.8 dB at an error probability level of $7{,}5 \cdot 10^{-9}$. The cosine channel of the modem T2 is practically inoperable at the value of the variable coefficient modem K_2 = 0.448, because for any signal-to-noise ratio, the error probability is 0.5. But with the value of the variable coefficient modem K_2 = 0.78, the cosine channel of the modem T2 becomes better than the sine channel.

Therefore, we will consider this modem to be bad and replace its code with another one: **modem T2*p*** (the letter p is a bad modem). The modem T2*p* does not have great prospects for existence in the future.

4.4.2. Noise immunity of modem T2 when receiving an additive mixture of noise and signal with the distribution of instantaneous values according to Tikhonov law with the parameter *D* = 2

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its cipher will be: **modem T2**. Let's repeat the analysis of modem noise immunity with the value of the Tikhonov distribution parameter *D* = 2. The modulation algorithm for a quasi-deterministic signal (2.33) is recorded in Table 4.34. The method and results of modem research are published in [33].

Table 4.34.

Signal modulation algorithm with V_m=1

Telegraph signal	**Signal dispersion value**	**The value of the expectation of the signal**
logical "0"	0,7645	0
logical "1"	0,7645	0,8

The results (4.22 – 4.25) need to be quantified. Tables 4.35 - 4.38 present the results of calculations at the values $\Pi_1 = 0{,}5$, $\Pi_2 = 0{,}7$, $K_1 = 0{,}5$, $K_2 = 0{,}715$, written in the line with the name of the evaluation.

Table 4.35.

Probability of errors in the cosine channel of the modem at a logical "1"

Threshold Π_{2k}	0,7·0,715 = 0,5					
Evaluation $\hat{A}(1,t)$	0	0	0,015	0,48	0,67	0,7
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	0,99	$2\cdot10^{-45}$	$2\cdot10^{-45}$

Table 4.36.

Probability of errors in the sinus channel of the modem at a logical "0"

Threshold Π_{1c}	0,50,5 = 0,25					
Evaluation $\hat{B}(1,t)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	$8\cdot10^{-32}$	$8\cdot10^{-32}$	$8\cdot10^{-32}$	$8\cdot10^{-32}$	$8\cdot10^{-32}$	$8\cdot10^{-32}$

Table 4.37.

Probability of errors in the sinus channel of the modem at a logical "1"

Threshold Π_{1c}	0,5·0,5 = 0,25					
Evaluation $\hat{B}(1,t)$	0	0	0	0,34	0,48	0,5
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$2,2\cdot10^{-5}$	$3,7\cdot10^{-27}$	$8\cdot10^{-32}$

Analysis of the data in Table 4.35 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors in the range of signal-to-noise ratios from 1 to 100 or from 0 dB to 20 dB. Table 4.36 presents the ideal results, as logical "0" in the sinus channel of the demodulator is determined without errors, i.e. with ultimate noise immunity, at any signal-to-noise power ratio. This allows us to say that simple control commands such as turn on-off, open-close and others will be accepted with a reliability equal to one, in any operating conditions of the modem T2.

Table 4.38.

Probability of errors in the cosine channel of the modem at a logical "0"

Threshold Π_{2k}	0,7·0,715 = 0,5					
Evaluation $\hat{A}(1,t)$	0	0	0,01	0,33	0,47	0,49
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	0	$2,2\cdot10^{-5}$	$1,6\cdot10^{-1}$

With the selected threshold values according to tables 4.37, 4.38, the distinction between logical "1" and zero in the sinus channel of the demodulator occurs without errors in the range of signal-to-noise ratios from 1 to 100, i.e. in the range equal to 20 dB. In this case, in the cosine channel of the demodulator, the maximum noise immunity is maintained at a signal-to-noise power ratio from 0.001 to 1, i.e. with a signal-to-noise ratio of 30 dB. If the signal-to-noise ratio is greater than one, there will be continuous errors in the cosine channel of the demodulator, i.e. operation of the modem in a noisy channel becomes impossible. And as a result of this, the cosine channel of the modem can be excluded from the structure of the demodulator, and only the sine channel can be left in the modem.

Let's move on from qualitative data analysis to a quantitative assessment of modem noise immunity. In tables 4.35 - 4.38, the following designations are adopted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - the total probability of device errors.

Quantitative assessment of the noise immunity of the modem T2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we get evaluations of the real and imaginary parts of the ch.f., which are recorded in tables 4.34 - 4.37. Both evaluations are random variables with their own properties and distribution laws. Repeating verbatim the rationale and methodology for calculating errors in the channels of the demodulator, set out in Section 4.1.1, we get the data recorded in Table 4.39.

The total error probability of the sine (curve 1,2,3) and cosine (curve 4,5,6) demodulator channel for different values of the parameter is shown in Figure 4.11. For comparison, in the same place from [15, p. 478] there is presented the error probability (curve 7) of ideal phase modulation (PM), calculated in a noisy channel. The main fragments of this noise immunity are listed in Table 4.39.

An analysis of the curves in Figure 4.11 confirms that the noise immunity of the sine and cosine channels of the demodulator is different and depends on the D - parameter of Tikhonov law. According to Table 4.39, it is desirable to take a large value of the parameter, and its optimal value can be obtained only as a result of constructing the theory of statistical modulation and conducting additional research. Comparison of the noise immunity of the new modem with the noise immunity of the known device, in which ideal PM is used, shows the superiority of its characteristics by at least ten orders, if we analyze weak signals, when $h^2 \leq 10$. In case of strong signals, when $h^2 > 10$, the noise immunity of an ideal PM is higher. The cosine and sine channels of the T2 modem are configured using the variable coefficients K_1, K_2 in different ways. In the cosine channel of the modem, the highest noise immunity at value $D = 5$ is obtained at ratio $h^2 = 1$ (curve 6), and with increasing value h^2 it decreases. This modem T2 channel is suitable for excellent performance with marginal noise immunity with weak signals. In the sinus channel of modem T2, the opposite is done (curve 3). The sinus channel of the modem T2 works equally well with both weak and strong signals. Therefore, we indicate in the modem T2 cipher that it is good. Its new cipher will be: **T2*x* modem**, where *x* is good.

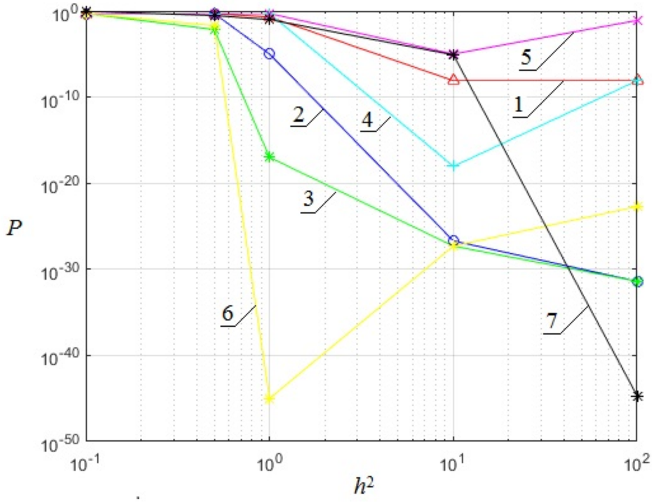


Figure 4.11. *The probability of errors of different models of the modem T2*

Thus, increasing the value of the parameter *D* of the Tikhonov distribution up two allows raising the thresholds Π_{1c}, $\Pi_{2к}$ at least twice in the sine and cosine channels of the modem T2 demodulator. And as a result of this, with a value of $h^2 = 1$, the probability of errors in the sinus channel of the T2 modem decreases by four orders of magnitude (Table 4.39)

Table 4.39.

Probability of errors of different modems

Sinus channel	*D*=1	Curve 1	0,5	0,5	$2\cdot10^{-1}$	$8\cdot10^{-9}$	$8\cdot10^{-9}$
	D=2	Curve 2	0,5	0,5	$1\cdot10^{-5}$	$2\cdot10^{-27}$	$4\cdot10^{-32}$
	D=5	Curve 3	0,5	$7\cdot10^{-3}$	$1\cdot10^{-17}$	$5\cdot10^{-28}$	$4\cdot10^{-32}$
Cosine channel	*D*=1	Curve 4	0,5	0,5	0,5	$9\cdot10^{-19}$	$8\cdot10^{-9}$
	D=2	Curve 5	0,5	0,5	0,5	$1\cdot10^{-5}$	$8\cdot10^{-2}$
	D=5	Curve 6	0,5	$2\cdot10^{-2}$	$1\cdot10^{-45}$	$5\cdot10^{-28}$	$2\cdot10^{-23}$
Ideal PM	Curve 7		0,9	$3\cdot10^{-1}$	$1\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$
Signal-to-noise ratio h^2			0,1	0,5	1	10	100

By increasing only the thresholds in the demodulator channels, the downward trend in the error probability in the sinus channel of the modem T2 continues up to the signal-to-noise ratio of 10 dB. In the cosine channel of the modem T2, unfortunately, such a trend is not observed. Nevertheless, such a model of the modem T2 is promising and can be included in the class of new generation modems.

4.4.3. Noise immunity of the modem T2 when accepting an additive mixture of noise and signal with the distribution of instantaneous values according to Tikhonov law with the parameter *D* = 5

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its cipher will be: **modem T2**. Let's repeat the analysis of the noise immunity of the modem with the value of the Tikhonov distribution parameter $D = 5$. The modulation algorithm for a quasi-deterministic signal (2.33) is recorded in Table 4.40. The method and results of modem research are published in [34].

Table 4.40.
Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,228	0
logical "1"	0,228	0,8

The results (4.22 – 4.25) need to be quantified. Tables 4.41 - 4.44 present the results of calculations at $\Pi_1 = 0{,}64$, $\Pi_2 = 0{,}9$, $K_1 = 0{,}625$, $K_2 = 0{,}78$, written in a line with the name of the evaluation.

Table 4.41.
Probability of errors in the cosine channel of the modem at a logical "1"

Threshold Π_{2k}	0,9·0,78 = 0,7					
Evaluation $\widehat{A}(1,t)$	0	0	0,29	0,81	0,89	0,9
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$2 \cdot 10^{-45}$	$2 \cdot 10^{-50}$	$2 \cdot 10^{-55}$

Table 4.42.

Probability of errors in the sinus channel of the modem at a logical "0"

Threshold $П_{1c}$	0,64·0,625 = 0,4					
Evaluation $\hat{B}(1)$	0	0	0	0	0	0
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	0	0	0

Table 4.43.

Probability of errors in the sinus channel of the modem at a logical "1"

Threshold $П_{1c}$	0,64·0,625 = 0,4					
Evaluation $\hat{B}(1)$	0	0	0,21	0,58	0,64	0,65
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$2,2·10^{-17}$	$1·10^{-27}$	$8·10^{-32}$

Table 4.44.

Probability of errors in the cosine channel of the modem at a logical "0"

Threshold $П_{2k}$	0,9·0,78 = 0,7					
Evaluation $\hat{A}(1)$	0	0	0,2	0,56	0,62	0,63
Relation h^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_0	0	0	0	$2·10^{-45}$	$1·10^{-27}$	$4,2·10^{-23}$

Analysis of the data in Table 4.41 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors in the range of signal-to-noise power ratios from 1 to 100 or from 0 dB to 20 dB. Table 4.42 presents the ideal results, as logical "0" in the sinus channel of the demodulator is determined without errors, i.e. with ultimate noise immunity, at any signal-to-noise power ratio. This allows us to say that simple control commands such as turn on-off, open-close and others will be accepted with a reliability equal to one, in any operating conditions of the modem T2.

With the selected threshold values according to the data of tables 4.43, 4.44, the distinction between logical "1" and zero in the sinus channel of the demodulator occurs without errors in the range of signal-to-noise ratios from 1 to 100, i.e. in the range equal to 20 dB. In this case, in the cosine channel of the demodulator, the maximum noise immunity is maintained at a signal-to-noise power ratio from 0.001 to 100, i.e. in the range of 50 dB.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the modem T2. In tables 4.41 - 4.44, the following designations are accepted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when acceptinga logical "1"; $P = \frac{1}{2}(P_0 + P_1)\cdot -$ total probability of device errors.

Quantitative assessment of the noise immunity of the modem T2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 4.40 - 4.43. Both estimates are random variables with their own properties and distribution laws. Repeating verbatim the rationale and methodology for calculating errors in the channels of the demodulator, set out in Section 4.1.1, we get the data recorded in Table 4.45.

The total error probability of the sine (curve 1) and cosine (curve 2) channels of the demodulator is shown in Figure 4.12. For comparison, in the same place from [15, p. 478] shows the error probability (curve 3) of ideal phase modulation (PM), calculated in a noisy channel.

Table 4.45.

Probability of errors of different modems

Total sinus channel error probability	0,5	$7{,}2\cdot10^{-3}$	$1{,}1\cdot10^{-17}$	$5\cdot10^{-28}$	$4\cdot10^{-32}$
Total cosine channel error probability	0,5	$1{,}7\cdot10^{-2}$	$1\cdot10^{-45}$	$5\cdot10^{-28}$	$2{,}1\cdot10^{-23}$
PM error probability	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$
Signal-to-noise ratio	0,1	0,5	1,0	10	100

Comparison of the noise immunity of the modem T2 with the noise immunity of the known device, in which ideal PM is used, shows the superiority of its characteristics by at least ten orders of magnitude. The cosine and sine channels of the modem T2 are configured using the variable coefficients K_1, K_2 in different ways. With the value D=5 in the cosine channel of the modem, the error probability itself is obtained with the ratio $h^2 = 1$ (curve 2), and with increasing value h^2, it grows. This modem T2 channel is suitable for excellent performance with marginal noise immunity with weak signals. In the sinus channel of the modem T2, the opposite is done (curve 1). The sinus channel of the modem T2 works equally well with both weak and strong signals. The probability of errors at a point $h^2 = 1$ in both modem channels is the same, and then curves 1,2 in Figure 4.12 diverge. Moreover, in the cosine channel of the modem, the error probability increases to the value $P=2.1\cdot10^{-23}$ with a signal-to-noise ratio of 20 dB, while in the sinus channel of the modem, the error probability continues to decrease to the value $P= 4\cdot10^{-32}$ with the same signal-to-noise ratio.

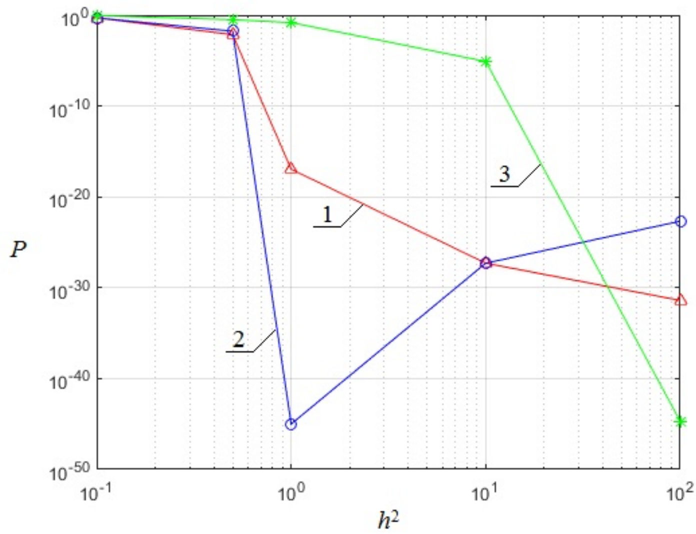


Figure 4.12. *Probability of modem T2 errors at D=5*

Thus, increasing the value of the parameter D of the Tikhonov distribution to five makes it possible to raise the thresholds $П_{1с}$, $П_{2к}$ at least three times in the sine and cosine channels of the modem T2 demodulator. And as a result of this, with a value $h^2 = 1$ by seventeen orders of magnitude (Table 4.39), the probability of errors in the sinus channel of the modem T2 decreases. By increasing only the thresholds in the demodulator channels, the downward trend in the error probability in the sinus channel of the modem T2 continues up to the signal-to-noise ratio of 20 dB. In the cosine channel of the modem T2, unfortunately, such a trend is not observed. Nevertheless, such a model of the modem T2 is promising and occupies a worthy place in the class of new generation modems.

Single-channel modem T2-1

Let the new T2-1 modem contain a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). The quasi-deterministic signal modulation algorithm (2.33) is written in Table 4.40. At the same time, the above analysis of the modem noise immunity when operating in a noisy channel remains unchanged for the new modem model. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of the execution of the truth table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator in Figure 3.7.

Table 4.42 shows that in the sinus channel of the demodulator, the logical "0" will be determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. Table 4.44 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, in practice this does not work out, which is confirmed by table 3.1 of truth. The probability of errors in the modem T2 - 1 is reduced by an average of 20 times compared with the error probability of the cosine channel of the modem T2.

The error probability of the modem T2 and the modem T2-1 is shown in Figure 4.13, where curve 1 refers to the T2-1 modem; curve 2 - to the cosine channel of modem T2; curve 3 - to the sinus channel of modem T2; curve 4 - to the device in which phase modulation is applied.

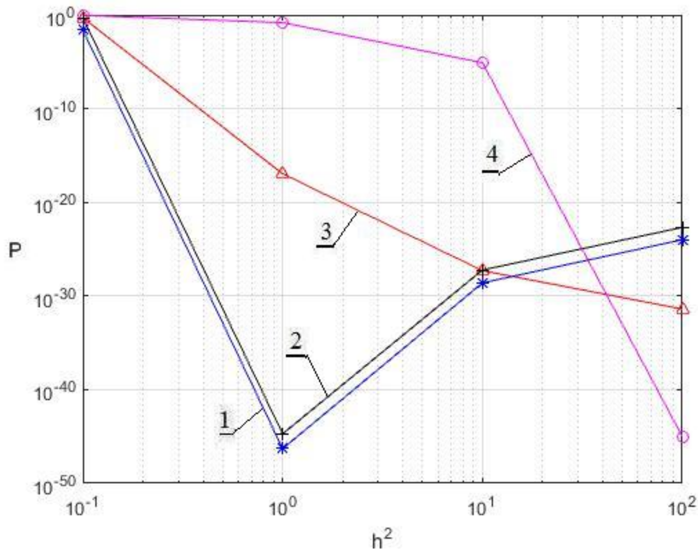


Figure 4.13. *Probability of modem errors T2 - 1*

An analysis of the graphs in Figure 4.13 shows that the modem T2-1 works well with weak signals and has an error probability of $1{\cdot}10^{-47}$ in the range of signal-to-noise ratios from 0.1 to 1.0 (minus 10 dB), and then the error probability starts to increase to the value of $1{\cdot}10^{-24}$ if the signal grows. After a signal-to-noise ratio of 10 dB, the modem T2-1 is equal in noise immunity to the sinus channel of the modem T2. However, both modems are superior in noise immunity to the device for receiving signals with ideal PM (curve 4) in the range of 26 dB, the lower limit of which is minus 10 dB. Starting from 16 dB to 20 dB, the modem T2 and the T2-1 modem together are inferior in terms of noise immunity to a device for receiving signals from PM. Let's recall that the modem T2 and the modem T2-1 use statistical modulation of a quasi-deterministic signal. Therefore, the modem T2 and the modem T2-1, when receiving weak signals, have no analogues and competitors all over the world. They can work with signals 10 times weaker than the noise power.

4.4.4. Noise immunity of the modem T1 when receiving an additive mixture of noise and signal with the distribution of instantaneous values according to Tikhonov law with a variable parameter *D*

The modem contains a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.9). Its name will be: **modem T1**. Let us repeat the analysis of the noise immunity of the modem, modulating only the distribution parameter of Tikhonov law. The modulation algorithm for a quasi-deterministic signal (2.33) is recorded in Table 4.46. The method and results of modem research are published in [16].

Table 4.46.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the distribution parameter of the Tikhonov law
logical "0"	1,604	1
logical "1"	0,228	5

Taking into account the modulation algorithm described above, we calculate the threshold in the demodulator in accordance with expression (3.13). As a result, with the value V_m = 1 and D = 5 we get

$$\Pi_1 = \frac{I_1(5)}{I_0(5)} = 0{,}89.$$

At the value V_m=1 we define for the additive mixture (4.21) the real part of the ch.f.

$$A(1,t) = \int_{-\infty}^{\infty} \cos(z) W(z) dz = \frac{I_1(D)}{I_0(D)} \exp\left(-\frac{\sigma_c^2}{2h^2}\right), \qquad (4.26)$$

where $h = \sigma_c / \sigma_ш$ - the signal-to-noise ratio. In expression (4.26) the signal dispersion $\sigma_c^{\,2}$ changes according to the modulation algorithm. Taking this into account, we write down the value of the estimate for the ch.f.

$$A(1,t) = \frac{I_1(5)}{I_0(5)} \exp\left(-\frac{\sigma_1^2}{2h_0^2}\right) \text{ at } s(t)=1 \text{ and } A(1,t) = \frac{I_1(1)}{I_0(1)} \exp\left(-\frac{\sigma_0^2}{2h_1^2}\right) \text{ at } s(t)=0, \qquad (4.27)$$

where $h_0 = {\sigma_0}/{\sigma_ш}$ - the signal-to-noise ratio when accepting a logical "0"; $h_1 = \sigma_1 / \sigma_ш$ - - signal-to-noise ratio when accepting a logical "1"; $\sigma_0^2 = 1{,}604$; $\sigma_1^2 = 0{,}228$.

The results (4.27) need to be quantified. Tables 4.47, 4.48 present the results of calculations at $П_1 = 0{,}89$, $K_1 = 0{,}84$, written in a line with the name of the evaluation. In this case the following is taken into account. The modulation algorithm in Table 4.46 contains $\sigma_0^2 = 1{,}604$ and $\sigma_1^2 = 0{,}228$. This means that at a constant noise power in the case of transmission of a logical "0" and a logical "1", the ratio $h_0^2 > h_1^2$. For the given values of the dispersions, we will get a value $h_0^2 = 7{,}035 h_1^2$ and use it in the calculations. When the modem is operating in a noisy channel, it is impossible to provide a different signal-to-noise ratio at its input when receiving a logical "0" and a logical "1", because the noise power in the channel does not depend on the logical "0" and "1". Therefore, the final conclusions about the probability of modem errors in this case should be taken depending on the values of the ratio h_1^2.

Analysis of the data in Table 4.47 shows that the modem T1 has maximum noise immunity when accepting a logical "1" in the range of signal-to-noise ratios from 1 to 100, i.e. in the range of 20 dB. From table 4.48 we see that when accepting a logical "0", the maximum noise immunity modem T1 takes place in the range of signal-to-noise ratios from 0.001 to 100, i.e., in the range of 50 dB. Let's remember that in the modem T1 the relation $\sigma_0^2 / \sigma_1^2 = 7{,}035$. So, when $\sigma_ш = const$, relation $h_0^2 > h_1^2$. Therefore, the final noise immunity characteristics of the modem T1 should be evaluated by errors when accepting a logical "1", i.e. in relation to h_1^2.

Table 4.47.

Probability of modem errors with logical "1"

Threshold $П_{1c}$	0,89·0,84 = 0,75					
Evaluation $\widehat{A}(1,t)$	0	0	0,29	0,81	0,89	0,9
Relation h_1^2	0,001	0,01	0,1	1,0	10	100
Probability of errors P_1	1	1	1	$2{,}2 \cdot 10^{-17}$	$2 \cdot 10^{-51}$	$2 \cdot 10^{-51}$

Table 4.48.

Probability of modem errors at logical "0"

Threshold $П_{1c}$	0,89·0,84 = 0,75					
Evaluation $\widehat{A}(1,t)$	0	0	0,143	0,4	0,442	0,446
Relation h_0^2	0,007	0,07	0,703	7,035	70,35	703,5
Probability of errors P_0	0	0	0	0	$2 \cdot 10^{-51}$	$2 \cdot 10^{-51}$

From a qualitative analysis of the data, let's move on to a quantitative assessment of the noise immunity of the modem T1. In tables 4.47, 4.48, the following designations are accepted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ – total probability of device errors.

Quantitative assessment of the noise immunity of the modem T1

In expression (3.11), an ideal adder is used instead of the expectation operator. And, as a result of this, we get an estimate of the real part of the ch.f., which is written in tables 4.47, 4.48. Evaluation of ch.f. is a random variable that has its own properties and distribution law. Repeating verbatim the rationale and methodology for calculating errors in the demodulator, set out in Section 4.1.1, we get the data recorded in Table 4.49.

The error probability of the modem T1 depending on the signal-to-noise ratio is shown using graphs in Figure 4.14, where curve 1 characterizes statistical modulation (SSK), and curve 2 characterizes QPSK modulation. For details on the error probability of the modem T1, its individual values are recorded in Table 4.49. For comparison, the error probability of a device for receiving signals with ideal phase modulation (QPSK) in the same place from [15, p.473] is given, calculated in a noisy channel.

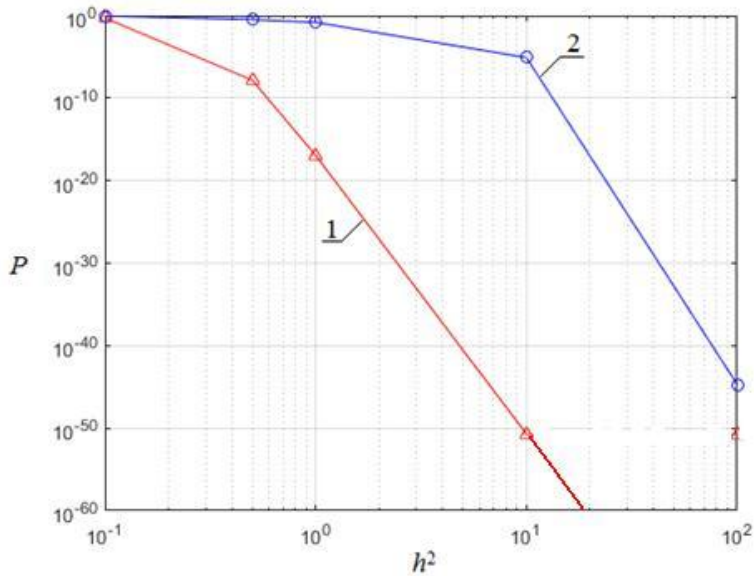


Figure 4.14. *Probability of modem T1 errors*

Table 4.49.

Probability of errors of different modems

Probability of modem T1 errors	0,5	$1{,}5 \cdot 10^{-8}$	$1{,}1 \cdot 10^{-17}$	$2 \cdot 10^{-51}$	Less than $2 \cdot 10^{-51}$
PM error probability	0,9	$3{,}2 \cdot 10^{-1}$	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$
Signal-to-noise ratio	0,1	0,5	1,0	10	100

Comparison of the noise immunity of the new modem T1 using SSK modulation with the noise immunity of the known device, which uses ideal PM, shows its superiority by at least ten orders and up to forty orders of magnitude. SSK modulation provides good noise immunity for weak signals. This effect arises due to the properties of ch.f. ideal to filter weak signals [7]. Therefore, at $h^2 = 0{,}5$ the error probability of the modem T1 is only $1.5 \cdot 10^{-8}$. In our opinion, with such data, we can hope for a good future for the modem T1.

Thus, the new modulation algorithm for a centered quasi-deterministic signal with the Tikhonov distribution made it possible to significantly improve the noise immunity of the modem T1 and made it possible to use it in a radio channel, since the expectation of the signal is always identically equal to zero. It turns out that a quasi-deterministic signal with the Tikhonov distribution law has advantages over other signals if only its dispersion is modulated. However, the demodulator then becomes single-channel (Fig. 3.9).

4.5. Performance indicators of digital systems with new generation modems

The class of quasi-deterministic signals includes oscillations of the form (2.1), (2.9), (2.19), (2.33) proposed by us, whose mathematical models do not contain time functions, but have only random variables. And, as a result of this, the energy spectra (2.8), (2.18), (2.32), (2.43) have one spectral line located at a frequency equal to the carrier frequency of the signal. In case **of statistical modulation** of signals (2.1), (2.9), (2.19), (2.33), their mathematical model changes and takes a different form (3.5), (3.8), which includes a telegraph signal *s(t)*, which is a sequence of elements in the form of logical "0" and logical "1". The elements of a telegraph signal carry information.

As an example, let's consider the signal (3.8). We assume that the telegraph signal has the Poisson distribution law and the correlation function $k_s(\tau)$, and it does not depend on the signal (2.9). Therefore, the signal correlation function (3.8), in accordance with its properties, will be equal to the sum of the correlation functions of independent variables [5]

$$k_{u_1}(\tau) = k_s(\tau) + k_u(\tau) = \frac{1}{4} e^2 \exp(-2\nu|\tau|) + k_u(\tau), \qquad (4.29)$$

Where ν - the average number of pulse jumps per unit time; τ - shift in time; e - amplitude of impulses. The signal correlation function (2.9) is defined in Chapter 2 and is equal to (2.16).

Let us proceed to the analysis of the power spectral density (energy spectrum) of the signal (3.8). Let us write the energy spectrum of the signal [16]

$$G_{u_1}(\omega) = \int_{-\infty}^{\infty} k_{u_1}(\tau)\exp(-j\omega\tau)d\tau = G_u(\omega) + G_s(\omega), \tag{4.30}$$

where [16]

$$G_s(\omega) = \frac{1}{2}e^2 \frac{\nu}{\nu^2 + \omega^2}. \tag{4.31}$$

the spectrum $G_u(\omega)$ was obtained and described earlier in Section 2.2. The spectrum (4.31) is continuous. The spectrum envelope has the form of a Gaussian curve at any carrier frequency of the numerical axis from -∞ to +∞. In the transition to the physical spectrum, i.e. to the spectrum in the region of positive frequencies, we obtain [16]

$$G_{u_1}(\omega) = \pi \times m_2\{a\}\delta(\omega + \omega_0) + \frac{e^2\nu}{\nu^2 + \omega^2}. \tag{4.32}$$

Let us determine the effective width of the energy spectrum of the modulated signal (3.8) at the frequency ω_0, after which we have [16]

$$\Delta\omega_э = \frac{1}{G_s(0)}\int_0^{\infty} G_s(\omega)d\omega = \frac{\pi\nu}{2}, \tag{4.33}$$

where $\Delta\omega_э = 2\pi\Delta F_э$, $\Delta F_э = \frac{\nu}{4}$. Expression (4.33) needs clarification. The fundamental provisions recorded in the book explain that ". . . the value ΔF can be interpreted as the width of a energy $\Delta\omega$ spectrum process uniform in the band, equivalent to the given one in terms of average power [4, p.202]". If the frequency band (4.33) is implemented in a digital communication system, then half of the power of the received signal will be lost. This is not allowed in practice. To preserve the total power of the received signal, one should fulfill the equality $\Delta\widehat{F}_э = 4\Delta F_э = \nu$, where $\Delta\widehat{F}_Э$ - the effective bandwidth of the digital system. Then the signal power loss will be $1.54 \cdot 10^{-6}$ %, i.e. practically zero.

Digital systems in the literature are characterized by performance indicators [35], which include noise immunity, spectral and energy efficiency.

Let's compare the potential capabilities of a digital system with a new generation modem, for example, a modem T1, and a digital system with a modem for accepting signals with ideal phase modulation, as the most promising among the known types of modulation. The potential noise immunity of a digital system with a modem T1 after calculations (Tables 4.47, 4.48) turns out to be limiting in a channel with "white" noise, i.e. the modem T1 has no errors when receiving data in the range of signal-to-noise ratios of 27 dB, starting from a ratio of minus 3 dB.

The potential noise immunity of a modem with ideal phase modulation (PM) is lower, and the error probability is greater by sixteen orders of magnitude. This is shown in Figure 4.14 and using the data in Table 4.49.

The spectral efficiency of digital systems with PM according to [36] is 2 [(bit/s)/Hz]. Theoretically, for modems with statistical modulation, it will be 4 [(bit / s) / Hz], if you use the formula [35]

$$\gamma = R_b / \Delta\hat{F}_Э, \tag{4.34}$$

where R_b =1000 - information transfer rate at binary coding, bit/s; $\Delta\hat{F}_Э$ - modem bandwidth at v = 1000 Hz. Taking into account the Nyquist bandwidth, the spectral efficiency of a statistic modulation modem can decrease to 2.8-3.6 [(bps)/Hz], but it is still greater than the spectral efficiency of digital PM systems.

The energy efficiency of digital systems with PM is calculated by the formula [35] with the same error probability in the channel with "white" noise in the 1 Hz band

$$\beta = R_b / \Delta\hat{F}_Э h^2 = \gamma / h^2, \tag{4.35}$$

where h^2 – the signal-to-noise power ratio. We choose the value of the error probability at the level of $1 \cdot 10^{-17}$ (fig. 4.14). Then for systems with PM at R_b = 1000 bit/s, the energy efficiency will be 0.13 [(bit/s)/Hz·dB]. For modems with statistical modulation, it will be 1.4 [(bit/s) / Hz·dB], i.e. it approaches the limit equal to $1/ \ln 2$ =1,44 [35]. Summing up the analysis of the effectiveness of digital systems with new generation modems, we see that these digital systems in all respects are approaching the limiting theoretical values of system efficiency known in the literature, and this confirms their promise.

4.6. Comparison of noise immunity of new generation modems

In total, we have considered 13 models of new generation modems. At the first stage of selection, 11 models of modems were left for further analysis, unfortunately, the T2*p* modem and the T2*x* modem were left without attention. For clarity, all other modem models are listed in Table 4.50. Let's pay attention, in accordance with the cipher in table 4.50, single-channel and dual-channel modems are recorded, which have sine and cosine channels independent of each other.

Table 4.50.

Models of new generation modems

New generation modems			
modem A1	modem B1		modem T1
modem A2	modem B2	modem K2	modem T2
modem A2–1	modem B2–1	modem K2–1	modem T2–1

Completing the analysis of the noise immunity of different variants of new generation modems, let us compare the error probabilities of modem A, modem B, modem K, modem T when operating in a channel with noise and signals that have different distribution laws. Such information is prepared separately for wired and radio communication channels. It is shown in Figure 4.15, where a) - radio - and wired communication channels; b) - wired communication channel.

In Figure 4.15a, curve 1 refers to modem B1, curve 2 - to modem T1, curve 3 - to modem A1 with the optimal modulation algorithm (Table 4.11), curve 4 - to the modem for receiving signals from PM. The main values of the error probability in Figure 4.15a are recorded in tables 4.15, 4.49.

It follows from the analysis of the graphs in Figure 4.15a that in the radio channel, preference should be given to modems for receiving modulated signals with distribution according to Veshkurtsev's law or according to Tikhonov's law. Moreover, Veshkurtsev law has priority out of the two named distribution laws, since the error probability of the modem B1 when accepting weak signals in the range $10^{-2} \le h^2 \le 1$ is less by 27 orders of magnitude than that of the modem T1 and the modem A1. In addition, the energy gain for the modem B1 compared to QPSK (curve 4) at the error probability level of $1 \cdot 10^{-45}$ is 30 dB, for the T1 modem it is only 12 dB, and for the modem A1 it does not exceed 10 dB.

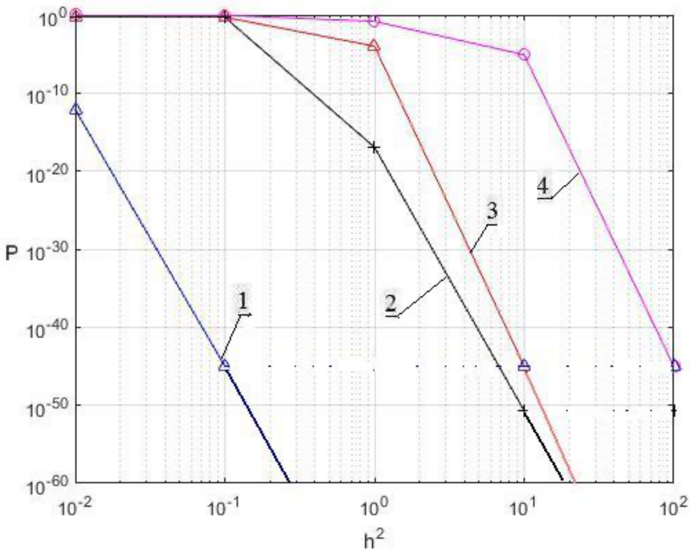


Figure 4.15a. *Error probability of single-channel modems in radio and wired communications*

In Figure 4.15b, curve 1 refers to the sinus channel of modem B2, curve 2 - to modem T2-1, curve 3 - to the sinus channel of modem A2, curve 4 - to the sinus channel of modem K2, curve 5 - to the modem for receiving signals from PM. The main values of the error probability in Figure 4.15b are presented in Table 4.51.

It follows from the analysis of the graphs in Figure 4.15b that modem B2 and modem T2–1 have a low probability of errors in the section $10^{-2} \le h^2 \le 10^0$. There is a big difference between modems in the area $1 \le h^2 \le 100$. Modem A2 becomes the leader in noise immunity, followed by modem B2, and then modem K2 goes and modem T2-1 closes the circuit. On the site $20 \le h^2 \le 100$ with modem A2, a modem competes for accepting signals from PM. In this section, the energy gain of the modem A2 compared to the QPSK modem is 10 dB. For weak signals, when $h^2 \le 10$, all new-generation modems have no analogues and competitors all over the world, and for the A2 modem, this can be said even with any signal-to-noise ratio.

Table 4.51.

Probability of errors of different modems

Sinus channel modem A2	0,5	0,5	$4\cdot10^{-32}$	$1\cdot10^{-45}$	Less than $1\cdot10^{-45}$
Sinus channel modem B2	$2,9\cdot10^{-2}$	$1,4\cdot10^{-26}$	$2,1\cdot10^{-31}$	$7,5\cdot10^{-33}$	$7,5\cdot10^{-33}$
Modem T2 - 1	$2,5\cdot10^{-2}$	$2,5\cdot10^{-2}$	$5\cdot10^{-47}$	$2,5\cdot10^{-29}$	$1\cdot10^{-24}$
Sinus channel of modem K2	0,5	0,5	$5,5\cdot10^{-13}$	$4\cdot10^{-32}$	$4\cdot10^{-35}$
Relation h^2	0,01	0,1	1,0	10	100

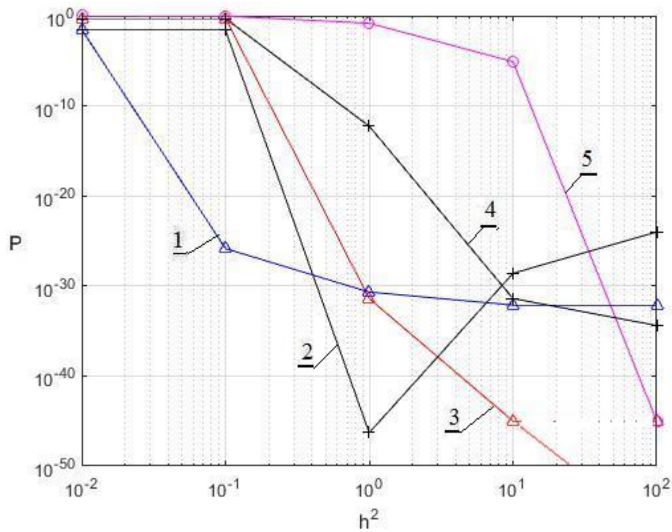


Figure 4.15b. *Probability of modem errors in wired communications*

In our opinion, the modem A2 will soon appear in digital systems with quadrature amplitude shift keying (QAM). Let's recall that the modem A2 is proposed for receiving signals with distribution according to the arcsine law. Such a distribution law has a physical process at the output of the generator, for example, electrical oscillations.

The rule follows from the history of the development of telegraphy on planet Earth that the decrease in the probability of modem errors by an order of magnitude while maintaining the signal strength occurs after ten years of exploratory scientific research in this direction. If you follow this rule, then the appearance of the modem B1 with an error probability of $1 \cdot 10^{-45}$ can be expected for more than three hundred years, counting from the present time, in which known PM modems have an error probability of $1 \cdot 10^{-8}$. This time will come, because technological progress cannot be stopped. In the meantime, the modems in table 4.51 are lined up in a certain queue one after another, the first in which, in our opinion, are modem B2 and modem A2.

5. NOISE IMMUNITY OF THE MODEM IN THE CHANNEL WITH "NON-WHITE" NOISE

"Non-white" noise has a Gaussian distribution law and, unlike "white" noise, it can have an expectation. For the operation of a new generation modem, such noise is dangerous, because significantly reduces its noise immunity. Let's consider the operation of each modem model separately in a channel with "non-white" noise or simply in a channel with Gaussian noise.

5.1.1. Noise immunity of the modem A2 when accepting an additive mixture of Gaussian noise and a non-centered signal with the distribution of instantaneous values according to the arcsine law

The modem contains a modulator (Fig. 3.1) and a two-channel demodulator (Fig. 3.7). Its cipher was recorded earlier as **modem A2**. Let's repeat the modulation algorithm for a quasi-deterministic signal (2.1) using Table 5.1.

Table 5.1.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,18	0
logical "1"	0,18	0,9

The analysis technique was developed in [26, 37]. Let's consider the noise immunity of a demodulator under the action of an additive mixture of a quasi-deterministic signal (2.1) and "non-white" Gaussian noise at its input

$$z(t)=u(t)+n(t), \qquad (5.1)$$

where *n(t)* – Gaussian noise, *u(t)* – a signal with a=U0 .

Using expressions (3.3, 3.4) and the data in Table 5.1, using formulas (3.13), we calculate the thresholds in the sine and cosine channels of the demodulator. As a result, with the value V_m = 1 and U_0 = 0,6 we get

$$\Pi_1=J_0(U_0,t)\sin(e_0) = 0{,}7116; \quad \Pi_2 = J_0(U_0,t) = 0{,}912.$$

Further, at the value V_m=1 we define for the additive mixture (5.1)

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z-e_ш)dz = J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\cos(e_ш)= \\ = J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos\left(\frac{e_0}{\rho}\right). \qquad (5.2)$$

When *s(t)*=0, then, similarly to (5.2), we calculate for the value V_m=1 for the additive mixture (5.1)

$$B(1,t)=\int_{-\infty}^{\infty}\sin(z)W(z-e_ш)dz=J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\sin(e_ш)=$$
$$=J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin\left(\frac{e_0}{\rho}\right), \quad (5.3)$$

where $W(z-e_ш)$ – the probability density of instantaneous values of the additive mixture; $h=\sigma_c/\sigma_ш$ – signal-to-noise ratio; $\sigma_c^2=U_0^2/2$ – the dispersion of the quasi-deterministic signal; $\sigma_ш^2$ – the dispersion of the Gaussian noise; $e_ш$ – the expectation of Gaussian noise; $\rho=e_0/e_ш$ the ratio of mathematical expectations of the signal and Gaussian noise (coefficient).

The results (5.2, 5.3) need to be quantified. Tables 5.2, 5.3 present the results of calculations at $П_1$=0,7116; $П_2$=0,912; K_1=0,56; K_2=0,88, e_0 = 0,9, written in the line with the name of the evaluation. In addition, in tables 5.2, 5.3, the values of the coefficient ρ are recorded in a separate column on the right.

Table 5.2.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold $П_{2k}$	0,912·0,88 = 0,8						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	0,23	0,52	0,57	0,57	1
Evaluation $\hat{A}(1,t)$	0	0	0,36	0,82	0,89	0,89	5
Evaluation $\hat{A}(1,t)$	0	0	0,37	0,83	0,91	0,91	10
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.3.

The values of the estimate of the ch.f. in the sinus channel of the modem

Threshold $П_{1c}$	0,7116·0,56 = 0,4						Coefficient ρ
Evaluation $\hat{B}$ (1,*t*)	0	0	0,29	0,65	0,71	0,71	1
Evaluation $\hat{B}(1,t)$	0	0	0,06	0,15	0,16	0,16	5
Evaluation $\hat{B}(1,t)$	0	0	0,03	0,07	0,08	0,08	10
Relation h^2	0,001	0,01	0,1	1,0	10	100	

In tables 5.2, 5.3, we compare the values of the ch.f. with the threshold specified in the first line. The values of the estimate depend on the coefficient ρ. Therefore, the noise immunity of the modem when operating in a channel with Gaussian noise will depend on two variables, namely, the signal-to-noise ratio h^2 and the coefficient ρ.

Analysis of the data in tables 5.2, 5.3 shows that there are errors in the sine and cosine channels of the demodulator when accepting a logical "0". When the value of the coefficient $\rho=1$, then continuous errors appear in the cosine channel of the modem when accepting a logical "0" for any signal-to-noise ratio. And, conversely, in the sinus channel of the modem, the maximum noise immunity is observed when accepting a logical "0" in the range of the signal-to-noise ratio of 20 dB, the lower limit of which is 0 dB. The situation changes dramatically when the value of the coefficient $\rho\geq5$. Now, in the cosine channel of the demodulator, the modem has the maximum noise immunity when operating in a channel with Gaussian noise when accepting a logical "0" in the range of signal-to-noise ratios of 20 dB, the lower limit of which is 0 dB. In this case, continuous errors are observed in the sinus channel of the demodulator when accepting a logical "0". Therefore, the modem has a maximum noise immunity when accepting a logical "0" in the range of signal-to-noise ratios of 20 dB, depending on the coefficient ρ, the values of which were indicated above. Moreover, the sine and cosine channels of the demodulator work in this case in different ways and there is no algorithm for choosing the preferred channel.

Suppose the additive mixture (5.1) contain a non-centered quasi-deterministic signal at the demodulator input, this corresponds to the condition *s(t)*=1. Similarly to (5.2), at the value $V_m=1$ we define

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z-e_0-e_ш)dz=J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\cos(e_0+e_ш)=$$
$$=J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos\left(\frac{e_0(1+\rho)}{\rho}\right) \quad (5.4)$$

or similarly (5.3) at the value $V_m=1$ let's calculate

$$B(1,t)=\int_{-\infty}^{\infty}\sin(z)W(z-e_0-e_ш)dz=J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\sin(e_0+e_ш)=$$
$$=J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin\left(\frac{e_0(1+\rho)}{\rho}\right). \quad (5.5)$$

The results (5.4), (5.5) require a quantitative analysis. Tables 5.4, 5.5 show calculation data at $П_1$=0,7116; $П_2$=0,912; K_1=0,56; K_2=0,88, written in the line with the name of the evaluation. In addition, in tables 5.4, 5.5, the values of the coefficient ρ are recorded in a separate column on the right.

Table 5.4.

The values of the estimate of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	0,912·0,88 = 0,8						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	-0,08	-0,19	-0,21	-0,21	1
Evaluation $\hat{A}(1,t)$	0	0	0,17	0,39	0,43	0,43	5
Evaluation $\hat{A}(1,t)$	0	0	0,2	0,45	0,5	0,5	10
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.5.

The values of the estimate of the ch.f. in the sinus channel of the modem

Threshold Π_{1c}	0,7116·0,56 = 0,4						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0	0	0,36	0,81	0,9	0,9	1
Evaluation $\hat{B}(1,t)$	0	0	0,33	0,73	0,8	0,8	5
Evaluation $\hat{B}(1,t)$	0	0	0,31	0,7	0,76	0,76	10
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Analysis of the data in tables 5.4, 5.5 shows that the accepting a logical "1" in the sine and cosine channels of the demodulator occurs without errors at any value of the coefficient ρ in the range of signal-to-noise ratios of 20 dB, the lower limit of which is different for each channel. For the sine channel of the demodulator, it is equal to 0 dB, and for the cosine channel, it is minus 30 dB. It turns out that the expectation of Gaussian noise does not affect the reception of the logical "1" by both channels of the modem. As a result, we can say that in the presence of Gaussian noise in the data transmission channel, the noise immunity according to Kotelnikov of the proposed modem is limiting, depending on the value of the expectation of noise. So, for example, at $\rho \geq 5$ and accurate synchronization of both channels of the modem, there are no errors when receiving a telegraph signal in the range of signal-to-noise power ratios of 20 dB, the lower limit of which is 0 dB.

Let's recall that the expectation of "non-white" Gaussian noise will be present in a wired communication channel and absent in a radio channel. In the radio channel, the effect of "white" and Gaussian noise on the operation of the modem is identical and was discussed earlier in Section 4.1. Therefore, the error probabilities of modem A2 in the channel with Gaussian noise at $e_ш = 0$ will be obtained from Table 4.6 and calculated for other values $e_ш$ using the method described in Section 4.1.1. The main values of the error probability of modem A2 are recorded in Table 5.6.

To visualize the error probability of the modem A2 depending on the signal-to-noise ratio and the value of ρ, the graphs in Figure 5.1 are presented. Curves 1 - 4 characterize the error probability of the sine channel, and curves 5 - 8 characterize the cosine channel of modem A2. Curve 9 shows the error probability of the device for accepting signals with ideal PM according to the work [15, p.473]. In Figure 5.1, curves 1.5 are the same for any signal-to-noise ratio. In addition, curves 7 and 8 also coincide with each other. This means that the expectation of "non-white" Gaussian noise at value does not affect the noise immunity of the cosine channel of modem A2, but positively affects the operation of the sine channel. In the sinus channel of modem A2 (curves 3 and 4), noise immunity increases by 10 dB. This means that the signal modulation algorithm in Table 5.1 is not optimal and can be corrected. Apparently, it would be more correct to write $e_0 = 1$. Then the probability of errors in the sinus channel of modem A2 at $h^2 = 0{,}1$ will decrease by five orders of magnitude up to the value P=1,1·10^{-5}. Moreover, nothing will change in the cosine channel of the modem A2.

And it's a completely different matter when the value of $e_ш \geq 0{,}1$. At $e_ш = 0{,}9$ For both modem channels, the error probability is 0.5 (curves 1.5) for any signal-to-noise ratio. Here, the noise immunity of the A2 modem in the channel with Gaussian noise reaches a minimum, as a result of which it becomes inoperable. To ensure the operation of the A2 modem with high noise immunity in a channel with Gaussian noise, additional measures are required. The contents of these activities are outlined below.

Table 5.6.

Probability of errors of different modems

P	0,5	0,5	0,4	0,5	0,5	0,9	Curve 1
P	0,5	5,5·10^{-4}	4·10^{-32}	5,5·10^{-30}	5,5·10^{-30}	0,45	Curve 2
P	0,5	1,1·10^{-5}	1·10^{-45}	1·10^{-45}	Less than 1·10^{-45}	0,09	Curve 3
P	0,5	0,5	4·10^{-32}	1·10^{-45}	Less than 1·10^{-45}	0	Curve 4
P	0,5	0,5	0,5	0,5	0,5	0,9	Curve 5
P	0,5	0,5	2,4·10^{-3}	2·10^{-37}	2·10^{-37}	0,45	Curve 6
P	0,5	0,5	1,1·10^{-5}	1·10^{-45}	Less than 1·10^{-45}	0,09	Curve 7
P	0,5	0,5	1,1·10^{-5}	1·10^{-45}	Less than 1·10^{-45}	0	Curve 8
P	0,9	3,2·10^{-1}	1,5·10^{-1}	8·10^{-6}	2·10^{-45}	0	Curve 9
h^2	0,01	0,1	1,0	10	100	$e_ш$	

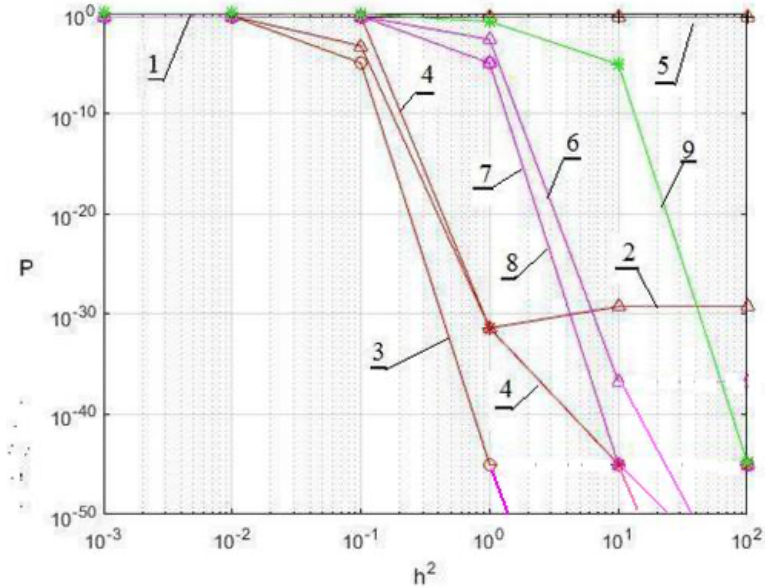


Figure 5.1. *Modem A2 Error Probability in a Channel with "Non-White" Gaussian Noise*

Single-channel modem A2-1

The new modem contains a modulator (Fig. 3.1) and a single-channel demodulator (Fig. 3.8). The modulation algorithm for a quasi-deterministic signal (2.1) remains the same and is recorded in Table 5.1. At the same time, the above theoretical analysis of the noise immunity of the modem A2 when operating in a channel with "non-white" Gaussian noise remains unchanged for the modem A2 - 1. However, the new model of the modem A2 has only one channel and one output, on which the telegraph signal will appear as a result of the states in table 3.1 of truth. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator shown in Figure 3.7.

The data in the table. 5.3 show that in the sinus channel of the demodulator, the logical "0" will be determined without errors. There are no errors in the sinus channel in the range of signal-to-noise power ratios equal to 50 dB, while the value of the coefficient $5 \le \rho \le 10$. Table 5.4 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors. There are no errors in the cosine channel in the range of signal-to-noise ratios equal to 50 dB, if the probability of errors $2 \cdot 10^{-45}$ is conditionally equated to zero. Combining these advantages of both channels together, we get a new modem model with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range being minus 30 dB. However, in practice this does not work, which is confirmed by truth table 3.1. The probability of errors in modem A2 - 1 decreases on average by 20 times compared with the probability of errors in the cosine channel of modem A2.

The probability of modem errors A2–1 is shown on fig. 5.2, where curve 1 is obtained with a coefficient ρ=1; curve 2 – with coefficient ρ=5; curve 3 – with coefficient ρ=10. In the same place, for comparison, the probability of errors of a known device (curve 5) using ideal phase modulation is shown. Curves 1,2,3,4 coincide in the section $10^{-3} \le h^2 \le 10^{-2}$ and curves 3,4 coincide at any value of h^2. The main values of the error probability of the modem A2-1 in the channel with "non-white" Gaussian noise are listed in Table 5.7.

Table 5.7.

Probability of errors of different modems

P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	0,9	Curve 1
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$1{,}2\cdot10^{-4}$	$1\cdot10^{-38}$	$1\cdot10^{-38}$	0,45	Curve 2
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$5{,}5\cdot10^{-7}$	$5\cdot10^{-47}$	Less than $5\cdot10^{-47}$	0,09	Curve 3
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$5{,}5\cdot10^{-7}$	$5\cdot10^{-47}$	Less than $5\cdot10^{-47}$	0	Curve 4
P	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$	0	Curve 5
h^2	0,01	0,1	1,0	10	100	$e_ш$	

An analysis of the curves in Figure 5.2 shows that the error probability of the A2-1 modem in a channel with “non-white” Gaussian noise strongly depends on the noise expectation value. If the value is $e_ш \le 0{,}1$, then the probability of modem errors A2 - 1 is minimal and lies at the level of $1\cdot10^{-47}$ and even lower. In this case, the probability of modem errors A2 - 1 increases to a value of $2.5\cdot10^{-2}$ (curve 1), as a result of which the noise immunity is completely lost. To restore the noise immunity of the A2-1 modem, it is necessary to compensate for the expectation of "non-white" Gaussian noise. Recommendations for eliminating the influence of non-white Gaussian noise characteristics on the noise immunity of new generation modems are outlined below.

Modem A2-1 in terms of noise immunity surpasses only the cosine channel of modem A2. It has a potential noise immunity in the range of 30 dB and in this indicator exceeds, at least twenty orders of magnitude, modems known from domestic and foreign literature. The A2-1 modem with such characteristics has no analogues and competitors all over the world.

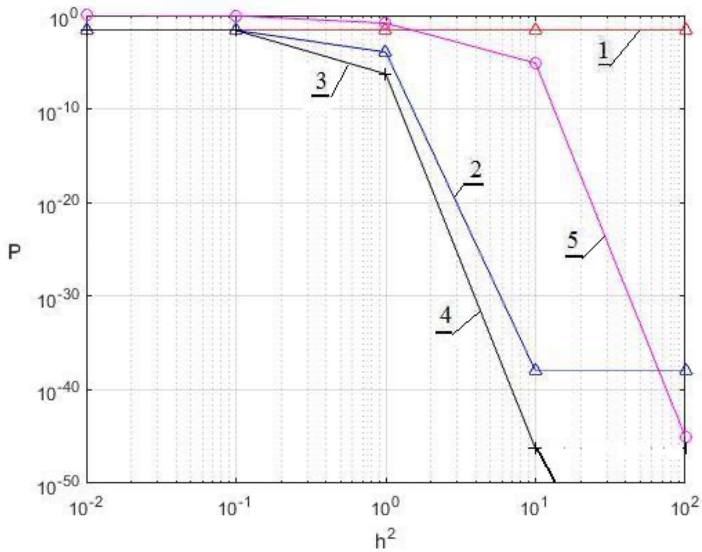


Figure 5.2. *Probability of modem A2–1 errors in a channel with "non-white" Gaussian noise*

Recommendations for eliminating the influence of the characteristics of "non-white" Gaussian noise

It is not possible to know the signal-to-noise ratio and the status of noise in the communication channel in advance before the communication session, namely: to consider the noise as "white" or "non-white" Gaussian. Unlike "white", other noise may have an expectation, which negatively affects the noise immunity of the demodulator (Fig. 3.7) of the modem A2 we propose. Therefore, additional measures are required to resolve this issue positively, leaving everything else unchanged. To this end, we recommend the following.

We propose to modernize the expressions (3.11, 3.12), bringing them to the form

$$\widehat{A}(V_m)=\frac{1}{N}\sum_{k=1}^{N}\cos\left[V_m\left[z(k\Delta t)-\widehat{m}_1\{n(t)\}\right]\right], \tag{5.6}$$

$$\widehat{B}(V_m)=\frac{1}{N}\sum_{k=1}^{\infty}\sin\left[V_m\left[z(k\Delta t)-\widehat{m}_1\{n(t)\}\right]\right] , \tag{5.7}$$

where $\widehat{m}_1\{n(t)\}$ - the estimate of the expectation of the noise. This expectation of noise is measured using the characteristic function in advance for no more than one second in the channel before the communication session, when there is still no signal, and is recorded in memory, and then, during modem operation, is subtracted from each current discrete instantaneous reading of the additive mixture (5.1) . Algorithm for measuring the expectation of a random process using ch.f. already developed [2]

$$\widehat{m}_1\{n(t)\} = \sum_{m=1}^{\infty}(-1)^{m+1}\left(\frac{2}{m\Delta V}\right)\widehat{B}(m\Delta V), \tag{5.8}$$

where

$$\widehat{B}(V_m) = \frac{1}{N}\sum_{k=1}^{N}\sin[V_m z(k\Delta t)] \tag{5.9}$$

the estimate of the imaginary part of the ch.f.; $V_m = m\Delta V$, ΔV - quantization step of the ch.f. The previously constructed theory for measuring estimates of probabilistic characteristics of random processes using ch.f. is described in the book [2], where the step ΔV is calculated and all the properties of estimates, including estimates (5.8,5.9), are studied. Moreover, a virtual device XN 31.1 *beta* has been developed, with which you can measure estimates of 15 probabilistic characteristics of a random process for no more than five seconds and thus control the status of noise and other interference. The description of the device and instructions for its use are published in the book [3]. We recommend to include separate files of the program of the device for measuring estimates (5.8,5.9) into the computer program of the modem A2 and thereby eliminate the influence of the numerical characteristics of noise (mathematical expectation) on the noise immunity of digital systems with amplitude shift keying.

5.1.2. Noise immunity of modem A1 when receiving an additive mixture of Gaussian noise and a centered signal with the distribution of instantaneous values according to the arcsine law

The modem contains a modulator (Fig. 3.2d) and a single-channel demodulator (Fig. 3.9). Its cipher was recorded earlier as **modem A1**. We repeat the modulation algorithm for a quasi-deterministic signal (2.1) using Table 5.8.

Table 5.8.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0	0
logical "1"	1,125	0

The analysis technique was developed in [37]. Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a centered quasi-deterministic signal (2.1) and “non-white” Gaussian noise acts at its input

$$z(t)=u(t)+n(t), \tag{5.10}$$

where $n(t)$ – Gaussian noise, $u(t)$ – a signal with $a=U_0$.

Using expressions (3.3, 3.4) and the data in Table 5.8, using formulas (3.13), we calculate the threshold in the cosine channel of the demodulator. As a result, at the value $V_m = 1$ and $U_0 =1{,}5$ we get

$$П_2 = J_0(U_0,t) = J_0(0) = 1.$$

Further, at the value $V_m=1$ and $s(t) =0$ we define for the additive mixture (5.10)

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z-e_ш)dz = J_0(U_0)\exp\left(-\frac{\sigma_ш^2}{2}\right)\cos(e_ш)=$$

$$= J_0\left(\sigma_c\sqrt{2}\right)\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(\rho e_ш), \tag{5.11}$$

where $W(z\text{-}e_ш)$ – the probability density of instantaneous values of the additive mixture; $h = \sigma_c/\sigma_ш$ – signal-to-noise ratio; $\sigma_c^2 = U_0^2/2$ – the dispersion of the quasi-deterministic signal; $\sigma_ш^2$ – the variance of the Gaussian noise; $e_ш$ – the expectation of noise; ρ – the coefficient. The result (5.11) needs to be analyzed quantitatively. Table 5.9 presents the results of calculations at $П_2=1$; $К_2=0{,}55$ and $\sigma_c^2 = 1{,}125$, $e_ш = 0{,}9$, written in the line with the name of the evaluation. In addition, in table 5.9, the values of the coefficient ρ are recorded in a separate column on the right.

Table 5.9.

The values of the evaluation of the ch.f. in the modem channel

Threshold $П_{1c}$	1·0,55 = 0,55						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	0,002	0,35	0,59	0,62	1
Evaluation $\hat{A}(1,t)$	0	0	0,004	0,51	0,86	0,9	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,004	0,57	0,95	0,99	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

In table 5.9, the values of the ch.f. compare with the threshold written in the first line. The noise immunity of the modem when operating in a channel with Gaussian noise now depends on two coefficients, namely, on the signal-to-noise ratio h^2 and on ρ, i.e. from the expectation of noise. Analysis of the data in Table 5.9 shows that there are errors in the modem A1 demodulator when accepting a logical "0". When the value of the coefficient is $0{,}5 \le \rho \le 1$,, then continuous errors appear in modem A1 when accepting a logical "0" at a signal-to-noise ratio from

10^{-3} to 10. The situation changes dramatically when the value of the coefficient $\rho < 0{,}5$. Now modem A1 has the maximum noise immunity when operating in a channel with Gaussian noise when accepting a logical "0" in the range of signal-to-noise ratios of 20 dB, the lower limit of which is 0 dB. Therefore, modem A1 has a maximum noise immunity when accepting a logical "0" in the range of signal-to-noise ratios of 20 dB, depending on the expectation of Gaussian noise, the values of which vary from 0 to 0.45.

In the range of signal-to-noise ratios from 0.1 to 1, errors are possible when receiving a logical "0". However, it can be stated that the noise immunity of the modem A1 when operating in a channel with Gaussian noise is an order of magnitude better than the data given in the publication.

Let the additive mixture (5.10) at the demodulator input contain a centered quasi-deterministic signal with dispersion $\sigma_c^2 = 1{,}125$, which corresponds to the condition *s(t)*=1. In this case, expression (5.11) will not change.

The result (5.11) needs to be analyzed quantitatively. Table 5.10 shows the calculation data at $П_2$=1; K_2=0,55 and $\sigma_c^2 = 1{,}125$, $e_ш = 0{,}9$, written in the line with the name of the evaluation. In addition, in table 5.10, the values of the coefficient ρ are recorded in a separate column on the right.

Similarly to the analysis of table 5.9, we will study the data in table 5.10. The data in Table 5.10 turned out to be below the set threshold at any value of the expectation of Gaussian noise. It turns out that the expectation of Gaussian noise does not affect the accepting the logical "1". Hence, they correspond to the ideal case. Therefore, it can be stated that the accepting the logical "1" in modem A1 occurs without errors (i.e. with maximum noise immunity) in the range of signal-to-noise ratios from 10^{-3} to 10^2 or in the range of 50dB. This data is like at least by three orders of magnitude better than the noise immunity of the modem, known from the publication. As a result, we can say that in the presence of "non-white" Gaussian noise in the data transmission channel, the potential noise immunity according to Kotelnikov of the proposed modem A1 is limiting, because with accurate modem synchronization, there are no errors when accepting a telegraph signal in the range of signal-to-noise ratios of 20 dB, and the lower limit of the range is 0 dB.

Table 5.10.

The values of the evaluation of the ch.f. in the modem channel

Threshold $П_{1c}$	1·0,55 = 0,55						Coefficient ρ
Evaluation $\widehat{A}(1,t)$	0	0	0,001	0,18	0,3	0,32	1
Evaluation $\widehat{A}(1,t)$	0	0	0,002	0,26	0,44	0,46	0,5
Evaluation $\widehat{A}(1,t)$	0	0	0,002	0,29	0,49	0,51	0,1

Relation h^2	0,001	0,01	0,1	1,0	10	100	

To visualize the demodulator error probability depending on the signal-to-noise ratio and the value of the coefficient ρ, the graphs are presented in Figure 5.3. The main values of the error probability of modem A1 are recorded in Table 5.11. Curves 1,2,3 characterize the noise immunity of the modem A1 according to the data obtained here, curve 4 - devices according to the data of [25], curve 5 - devices for receiving signals with ideal PM according to the data of [15, p.473]. In figure 5.3 curves 3,4 coincide. This means that the expectation of Gaussian noise at value $e_ш \leq 0{,}1$ does not affect the noise immunity of the modem A1.

And it's a completely different matter when the value is $e_ш > 0{,}1$. Here, the noise immunity of modem A1 in a channel with Gaussian noise depends significantly on the signal-to-noise ratio. For strong signals, $h^2 \geq 10$ the noise immunity of the modem A1 increases by an order of magnitude or more, and for weak signals $h^2 \leq 1$ it drops sharply. Let us determine, starting from the value $h^2 = 1$ using curve 1, the decrease noise immunity of the modem in the channel with "non-white" Gaussian noise in comparison with its characteristic obtained earlier (curve 4). It turned out to be 5 dB. Therefore, additional measures are needed to eliminate the influence of the expectation of Gaussian noise on the noise immunity of the modem A1. These activities are outlined in Section 5.1.1.

Table 5.11.

Probability of errors of different modems

P	0,5	0,5	0,5	$7{,}7\cdot10^{-9}$	$2{,}1\cdot10^{-23}$	0,9	Curve 1
P	0,5	0,5	0,5	$1{,}1\cdot10^{-45}$	$2\cdot10^{-35}$	0,45	Curve 2
P	0,5	$5\cdot10^{-1}$	$2{,}5\cdot10^{-3}$	$1{,}1\cdot10^{-17}$	$7{,}5\cdot10^{-9}$	0,09	Curve 3
P	0,5	$5\cdot10^{-1}$	$2{,}5\cdot10^{-3}$	$1{,}1\cdot10^{-17}$	$7{,}5\cdot10^{-9}$	0	Curve 4

P	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$	0	Curve 5
h^2	0,01	0,1	1	10	100	$e_ш$	

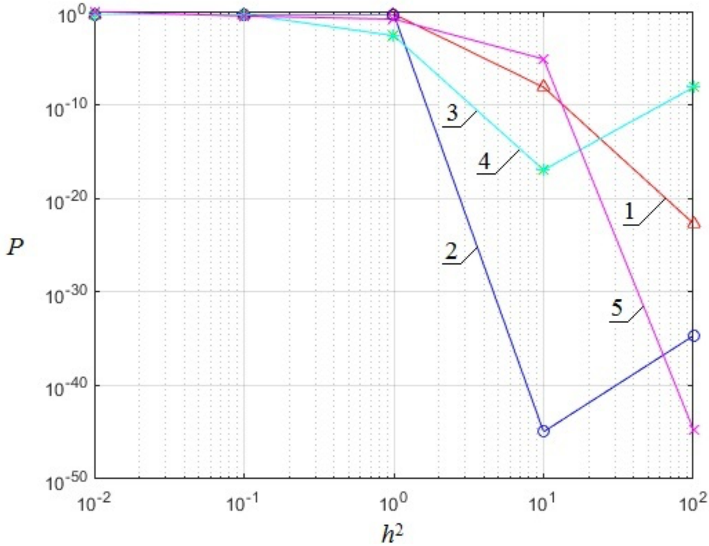


Figure 5.3. *Modem A1 error probability in a channel with "non-white" Gaussian noise*

5.2.1. Noise immunity of the modem B2 when receiving an additive mixture of Gaussian noise and a signal with the distribution of instantaneous values according to the non-centered Veshkurtsev law

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its cipher was recorded earlier as **modem B2**. The modulation algorithm for a quasi-deterministic signal (2.9) is repeated in Table 5.12.

Table 5.12.

Signal modulation algorithm with $V_m=1$

Telegraph signal	**Signal dispersion value**	**The value of the expectation of the signal**
logical "0"	0,01	0
logical "1"	0,01	0,6

The research methodology and results are published in [10,13,31]. Let us turn to the analysis of the noise immunity of the modem B2 under the action of an additive mixture of a quasi-deterministic signal (2.9) and "non-white" Gaussian noise at its input

$$z(t)=u(t)+n(t), \qquad (5.12)$$

where *n(t)* – the Gaussian noise, *u(t)* – signal (2.9).

Using expressions (2.12,3.10) and the data in Table 5.12, using formulas (3.13), we calculate the thresholds in the demodulator. As a result, at the value $V_m = 1$ we get

$$\Pi_1 = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right)\sin(e_0) = 0{,}5646\,, \quad \Pi_2 = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right) = 1.$$

Let us calculate the real and imaginary parts of the ch.f. of additive mixture (5.12) and is comparable with the thresholds. Then, during the transmission *s(t)* = 0 and value $V_m = 1$ in the channels of the demodulator (Fig. 3.7), the threshold devices will receive the values of the real and imaginary parts of the ch.f. additive mixture, equal to

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\cos(\rho e_ш), \qquad (5.13)$$

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\sin(\rho e_ш), \qquad (5.14)$$

where $W(z)$ - the probability density of the additive mixture; $h = \sigma_0/\sigma_ш$ - signal-to-noise ratio; $\sigma_ш^2$ - variance of Gaussian noise; $e_ш$ - expectation of Gaussian noise; ρ – the coefficient. When transmitting *s(t)* = 1 and the value $V_m = 1$ in the channels of the demodulator (Fig. 3.7), the threshold devices will receive the values of the real and imaginary parts of the ch.f. of additive mixture, equal to

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z+e_0)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\cos(e_0+\rho e_ш), \qquad (5.15)$$

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z+e_0)dz = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h^2}{4h^2}\right)\right]\sin(e_0+\rho e_ш). \qquad (5.16)$$

Suppose $K_2 = 0{,}96; K_1 = 0{,}532; \Pi_2 = 1; \Pi_1 = 0{,}5646; \sigma_0^2 = 0{,}01; e_0 = 0{,}6; e_ш = 0{,}6$. The results of calculations by formulas (5.13 - 5.16) are summarized in tables 5.13 - 5.16, recorded in the line with the name of the assessment. In addition, in tables 5.13 - 5.16, the values of the coefficient ρ are recorded in a separate column on the right.

An analysis of the data in tables 5.13 - 5.16 shows that modem B2 is very sensitive to the mean of Gaussian noise. For example, logical "0" in the sine channel and in the cosine channel of modem B2 is determined with errors if the expectation of Gaussian noise lies within $0{,}06 < e_ш \le 0{,}6$. But the logical "1" in the cosine channel and in the sine channel of modem B2 is determined correctly, i.e. without errors, for any value of the expectation in the range $0 \le e_{ш} \le 0{,}6$. An analysis of the data in tables 5.13 - 5.16 shows that modem B2 is very sensitive to the mean of Gaussian noise. For instance, logical "0" in the sine channel and in the cosine channel of modem B2 is determined with errors if the expectation of Gaussian noise lies within $0{,}06 < e_{ш} \le 0{,}6$. But the logical "1" in the cosine channel and in the sine channel of modem B2 is determined correctly, i.e. without errors, for any value of the expectation in the range $0 \le e_{ш} \le 0{,}6$.

Table 5.13.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	1·0,96 = 0,96						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0,0055	0,5	0,785	0,817	0,8253	0,8253	1
Evaluation $\hat{A}(1,t)$	0,0064	0,58	0,91	0,95	0,9553	0,9553	0,5
Evaluation $\hat{A}(1,t)$	0,0067	0,61	0,95	0,99	0,9982	0,9982	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.14.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	1·0,96 = 0,96						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0,0024	0,22	0,345	0,359	0,362	0,362	1
Evaluation $\hat{A}(1,t)$	0,0042	0,377	0,591	0,615	0,622	0,622	0,5
Evaluation $\hat{A}(1,t)$	0,0053	0,479	0,751	0,782	0,79	0,79	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.15.

The values of the estimate of the ch.f. in the sinus channel of the modem

Threshold Π_{1c}	0,532·0,5646= 0,3						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0,0038	0,342	0,537	0,559	0,565	0,565	1
Evaluation $\hat{B}(1,t)$	0,002	0,179	0,281	0,293	0,296	0,296	0,5

Evaluation $\hat{B}(1,t)$	0,0004	0,036	0,057	0,059	0,06	0,06	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.16.

The values of the estimate of the ch.f. in the sinus channel of the modem

Threshold Π_{1c}	0,532·0,5646 = 0,3						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0,0062	0,565	0,887	0,923	0,932	0,932	1
Evaluation $\hat{B}(1,t)$	0,0052	0,475	0,745	0,775	0,783	0,783	0,5
Evaluation $\hat{B}(1,t)$	0,0041	0,372	0,583	0,607	0,613	0,613	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

In this modem, the sinus channel prevails, since it has a maximum noise immunity in a wide range of signal-to-noise power ratios of 50 dB when operating in a channel with Gaussian noise, and the lower limit of the range is minus 30 dB.

From a qualitative analysis of the data, let's move on to a quantitative assessment of the noise immunity of the B2 modem. For this, the following designations are accepted: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ – total probability of device errors.

Quantitative assessment of modem noise immunity B2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 5.13 - 5.16. Both estimates are random variables with their own properties and distribution laws. Repeating verbatim the rationale and methodology for calculating errors in the demodulator channels (Fig. 3.7), set out in Section 4.1.1, we obtain the data recorded in Table 5.17. For comparison, in the same place from [15, p.473], the error probability of a device for receiving signals with ideal phase modulation (PM), calculated in a noisy channel, is given.

The dependence of the modem B2 error probability on the signal-to-noise ratio in the channel with Gaussian noise is shown in Figure 5.4, where curves 1-4 refer to the modem's sine channel; curves 5 - 8 - to the cosine channel of the modem; curve 9 - to the device for receiving signals with ideal PM. Curves 1,5,6 coincide at any value of h^2, and in the range of values, $10^{-1} \le h^2 \le 10^2$ curve 2 joins them. Curves 7,8,9 merge with curves 1,5,6 in the section $10^{-3} \le h^2 \le 10^{-1}$.

An analysis of the curves in Figure 5.4 shows that the modem B2 error probability in a channel with "non-white" Gaussian noise is at the level of 0.5 when the noise expectation is large and lies within $0{,}06 < e_{ш} \le 0{,}6$. Here, there is no need to talk about any noise immunity of the modem B2, since it becomes unable to receive digital data. To restore the noise immunity of modem B2, it is necessary to compensate for the expectation of Gaussian noise. Recommendations for the implementation of such an activity are developed and presented in section 5.1.1. Following them it is enough to reduce the expectation of noise to the value $e_{ш} \leq 0{,}06$ (ρ=0,1). Then the noise immunity of modem B2 will be practically the same as that considered earlier in the channel with "white" noise. In Figure 5.4, curves 3,4 and curves 7,8 confirm this.

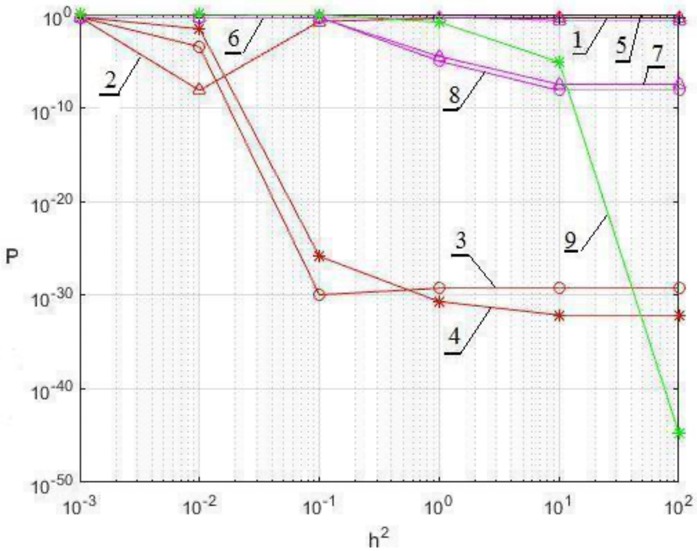


Figure 5.4. *Modem B2 error probability in a channel with "non-white" Gaussian noise*

Table 5.17.

Probability of errors of different modems

P	0,5	0,5	0,4	0,5	0,5	0,6	Curve 1
P	$7{,}5\cdot10^{-9}$	0,18	0,38	0,42	0,42	0,3	Curve 2
P	$3{,}5\cdot10^{-4}$	$1{,}1\cdot10^{-30}$	$5{,}5\cdot10^{-30}$	$5{,}5\cdot10^{-30}$	$5{,}5\cdot10^{-30}$	0,06	Curve 3
P	$2{,}9\cdot10^{-2}$	$1{,}4\cdot10^{-26}$	$2{,}1\cdot10^{-31}$	$7{,}5\cdot10^{-33}$	$7{,}5\cdot10^{-33}$	0	Curve 4
P	0,5	0,5	0,5	0,5	0,5	0,6	Curve 5
P	0,5	0,5	0,5	0,25	0,25	0,3	Curve 6
P	0,5	0,43	$3{,}3\cdot10^{-5}$	$3{,}4\cdot10^{-8}$	$3{,}4\cdot10^{-8}$	0,06	Curve 7
P	0,5	0,5	$1{,}1\cdot10^{-5}$	$7{,}5\cdot10^{-9}$	$7{,}5\cdot10^{-9}$	0	Curve 8
P	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$	0	Curve 9

h^2	0,01	0,1	1,0	10	100	$e_ш$

Again, in modem B2, the sine channel prevails over the cosine channel. Up to a signal-to-noise ratio of 16 dB, this channel is better in terms of noise immunity than a device for receiving signals with ideal PM (curve 9), and for the cosine channel of modem B2, this superiority remains only up to a ratio of 10 dB (curves 7,8).

Single-channel modem B2-1

The new modem contains a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). The modulation algorithm for a quasi-deterministic signal (2.9) is the same as before and is written in Table 5.12. At the same time, the above theoretical analysis of the modem B2 noise immunity in a channel with "non-white" Gaussian noise remains unchanged for the new modem model. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of the coincidence of the channel states recorded in the truth table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator in Figure 3.7.

Table 5.15 shows that in the sinus channel of the demodulator, the logical "0" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB, if there is the inequality $e_ш < 0{,}06$. Table 5.14 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, in practice this does not work, which is confirmed by truth table 3.1. The probability of errors in modem B2 - 1 is reduced by an average of 20 times compared with the error probability of the cosine channel of modem B2.

The error probability of the new generation B2-1 modem is shown in Figure 5.5, where curve 1 is plotted for the value $e_ш$ =0,6; curve 2 - for the value $e_ш$ =0,3; curve 3 - for the value $e_ш$ =0,06; curve 4 - for the value $e_ш$ =0. It also shows the error probability of a known device (curve 5) for receiving signals with phase modulation. The main values of the B2-1 modem error probability in a channel with Gaussian noise are recorded in Table 5.18.

Table 5.18.

Probability of errors of different modems

P	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	0,6	Curve 1
P	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	$2{,}5 \cdot 10^{-2}$	0,3	Curve 2
P	$2{,}5 \cdot 10^{-2}$	$2{,}1 \cdot 10^{-2}$	$1{,}6 \cdot 10^{-6}$	$1{,}7 \cdot 10^{-9}$	$1{,}7 \cdot 10^{-9}$	0,06	Curve 3

P	0,5	0,5	$1,1 \cdot 10^{-5}$	$7,5 \cdot 10^{-9}$	$7,5 \cdot 10^{-9}$	0	Curve 4
P	0,9	$3,2 \cdot 10^{-1}$	$1,5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$	0	Curve 5
h^2	0,01	0,1	1,0	10	100	$e_ш$	

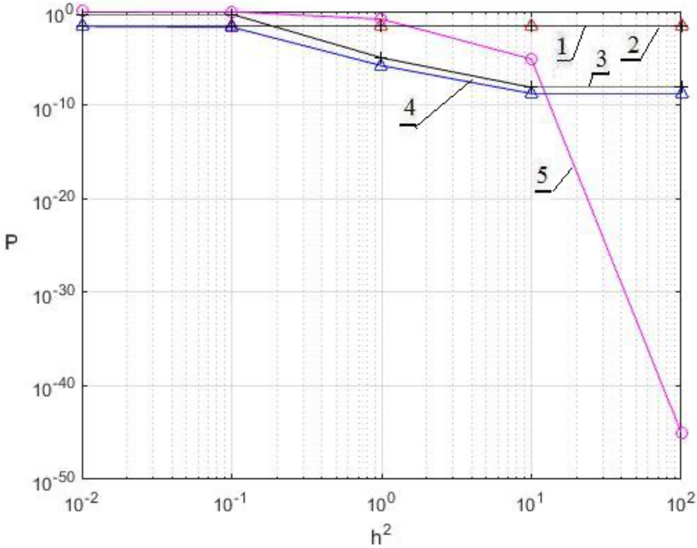


Figure 5.5. *Modem B2-1error probability in a channel with "non-white" Gaussian noise*

An analysis of curves 1,2,3 in Figure 5.5 shows that the modem B2-1 error probability is highly dependent on the expected value of non-white Gaussian noise over the 40 dB signal-to-noise ratio range. Only when $e_ш$ =0 the error probability stabilizes at the level of $7.5 \cdot 10^{-9}$ in the range of the signal-to-noise ratio $10 \le h^2 \le 100$. Therefore, the modem B2-1 has no potential noise immunity in a channel with "non-white" Gaussian noise, in which the expectation is present and changing. Recommendations for compensation of the mathematical expectation of "non-white" Gaussian noise in a wired channel are developed and presented in Section 5.1.1. The modem B2-1 is inferior in noise immunity to the sinus channel of the modem B2 by more than twenty orders of magnitude when working with "non-white" Gaussian noise.

5.2.2. Noise immunity of modem B1 when receiving an additive mixture of Gaussian noise and a signal with the distribution of instantaneous values according to the centered Veshkurtsev law

The modem contains a modulator (Fig. 3.3) and a single-channel demodulator

(Fig. 3.9). Its cipher was recorded earlier as **modem B1**. The modulation algorithm for a quasi-deterministic signal (2.9) is repeated in Table 5.19.

Table 5.19.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,0009	0
logical "1"	1,0	0

The research methodology and results are published in [10,13,31]. Let us turn to the analysis of the noise immunity of the demodulator, when an additive mixture of a quasi-deterministic signal (2.9) and "non-white" Gaussian noise with expectation acts at its input

$$z(t)=u(t)+n(t), \tag{5.17}$$

where $n(t)$ –the Gaussian noise, $u(t)$ – signal (2.9) .

Using expression (2.12) and the data in Table 5.19, using formula (3.13), we calculate the threshold in the demodulator. As a result, at the value V_m = 1 we get

$$\Pi1 = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left(-\frac{\sigma_0^2}{4}\right) = 0{,}7917.$$

Let us represent the functional transformation in the demodulator circuit (Fig. 3.9) by the dependence $y = \cos z$ at the value V_m = 1 and N>> 1. Calculate the mathematical expectation $m_1\{y\}$, since the ch.f. is the expectation of the cosine function for the real part and the sine function for the imaginary part. Let's recall that the imaginary part of the ch.f. signal (2.9) is equal to zero. We get at the value V_m =1

$$m_1\{y\} = \int_{-\infty}^{\infty}\cos(z)W(z-e_ш)dz = I_0\left(\frac{1}{4}\sigma^2\right)\exp\left[-\left(\frac{\sigma^2+2\sigma_ш^2}{4}\right)\right]\exp\left(je_ш\right), \tag{5.18}$$

where $W(z-e_ш)$ - the probability density of the additive mixture (5.20); $\sigma_ш^2$ - dispersion of Gaussian noise; $e_ш$ – the expectation of the Gaussian noise. Dispersion of the modulated c.c.s. varies discretely from σ_0^2 to σ_1^2, the values of which are recorded in Table 5.19. Then, when transmitting a logical "0", we get

$$m_1\{y\}_0 = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\left(\frac{\sigma_0^2+2\sigma_ш^2}{4}\right)\right]\cos\left(e_ш\right), \tag{5.19}$$

and when transmitting a logical "1" we will have

$$m_1\{y\}_1 = I_0\left(\frac{\sigma_1^2}{4}\right)\exp\left[-\left(\frac{\sigma_1^2+2\sigma_ш^2}{4}\right)\right]\cos\left(e_ш\right). \tag{5.20}$$

Having performed the following substitutions in expressions (5.19,5.20) $\sigma_{ш}^{2} = \sigma_{0}^{2} / h_{0}^{2}$, $\sigma_{ш}^{2} = \sigma_{1}^{2} / h_{1}^{2}$, we get

$$\hat{A}(1,t) = I_0\left(\frac{\sigma_0^2}{4}\right)\exp\left[-\sigma_0^2\left(\frac{2+h_0^2}{4h_0^2}\right)\right]\cos(\rho e_{ш}), \tag{5.21}$$

$$\hat{A}(1,t) = I_0\left(\frac{\sigma_1^2}{4}\right)\exp\left[-\sigma_1^2\left(\frac{2+h_1^2}{4h_1^2}\right)\right]\cos(\rho e_{ш}), \tag{5.22}$$

where $h_0=\sigma_0 / \sigma_{ш}$ - signal-to-noise ratio when receiving logical "0"; $h_1= \sigma_1 / \sigma_{ш}$ - signal-to-noise ratio when receiving a logical "1"; ρ – coefficient. The results (5.21), (5.22) require a quantitative analysis. Tables 5.20,5.21 show calculation data at $V_m=1$, $\sigma_1=0{,}03$, $\sigma_0= 1$, $\Pi_{1c} = 0{,}9$,

Table 5.20.

The values of the evaluation of the ch.f. in the modem channel

Threshold Π_{1c}	0,7917·1,14 = 0,9						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0,444	0,663	0,7	0,7	0,7	0,7	1
Evaluation $\hat{A}(1,t)$	0,59	0,88	0,92	0,92	0,92	0,92	0,5
Evaluation $\hat{A}(1,t)$	0,635	0,948	0,997	0,997	0,997	0,997	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.21.

The values of the evaluation of the ch.f. in the modem channel

Threshold Π_{1c}	0,7917·1,14 = 0,9						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0,352	0,527	0,549	0,552	0,552	0,552	1
Evaluation $\hat{A}(1,t)$	0,465	0,697	0,726	0,73	0,73	0,73	0,5
Evaluation $\hat{A}(1,t)$	0,503	0,755	0,785	0,789	0,789	0,789	0,1
Relation h^2	1,11111	11,1111	111,111	1111,11	11111,1	111111	

$e_{ш} = 0{,}8$, written in a line with the name of evaluation. In addition, in tables 5.20, 5.21, the values of the coefficient ρ are recorded in a separate column on the right.

When the modem is operating in a noisy channel, it is impossible to know the different signal-to-noise ratio at its input when accepting a logical "0" and a logical "1", because the noise power in the channel does not depend on the telegraph signal. In our example, the signal dispersions during the transmission of telegraph signal elements correlate with each other as $\sigma_0^2 / \sigma_1^2 = 1111,11$. In this regard, in a noisy channel, the ratio is $h_0^2 = 1111,11 h_1^2$ at constant noise power.

In Table 5.21, all evaluation values $\hat{A}(1,t)$ are less than the threshold for any signal-to-noise ratio, regardless of the coefficient ρ. This means that in a channel with "non-white" Gaussian noise, modem B1 does not have errors when accepting a logical "0" in the range of signal-to-noise ratios of 50 dB. In Table 5.20, the evaluation values $\hat{A}(1,t)$ exceed the threshold for a signal-to-noise ratio of 0.1 to 100 when the coefficient is $0 \le \rho \le 0,5$. . Here, in a channel with Gaussian noise, modem B1 has no errors when accepting a logical "1". At $h_1^2 < 0,1$ and the value of the coefficient ρ = 0,5 of the demodulator (Fig. 3.10), errors appear in the channel with Gaussian noise when accepting a logical "1". Thus, at a value ρ = 0,5 the signal-to-noise ratio range is only 30 dB. If the value is $\rho \le 0,1$, then the range of signal-to-noise ratios for modem B1 increases to 40 dB. It turns out that the expectation of "non-white" Gaussian noise affects the noise immunity of modem B1.

The final conclusions about the noise immunity of the modem will be made according to the data in Table 5.20 (the accepted designations h_0^2, h_1^2 are further simplified to the form h^2). Its analysis shows that the noise immunity of modem B1 will be the limit in the range of signal-to-noise power ratios from 0.01 to 100, i.e. in the range of 40 dB, if the coefficient is $\rho \le 0,1$. This indicates that the expectation operator in the mathematical model of ch.f. reliably protects the signal from Gaussian noise. New generation modems can work without errors when the signal-to-noise ratio is less than one.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the B1 modem. Let's introduce the following designations: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - is the total probability of device errors.

Quantitative assessment of modem noise immunity

The demodulator (Fig. 3.10) measures the value of the real part of the ch.f. with some error. And, as a result of this, we obtain $\hat{A}(1,t)$ - an estimate of the real part of the ch.f. Estimated ch.f. is a random variable that has its own properties and distribution law. Repeating verbatim to the conditions of our problem the methodology for calculating errors in the demodulator, written in detail in Section 4.1.1, we obtain the data included in Table 5.22. Curve 1 was obtained at a value of ρ =0,5, curve 2 - at a value of ρ = 0.1, curve 3 - at a value of ρ =0. For comparison, in the same place from [15, p.473], the probability of errors of ideal phase modulation is given (curve 4).

Table 5.22.

Probability of errors of different modems

Curve 1	0,5	0,5	$2{,}3\cdot10^{-3}$	$2{,}3\cdot10^{-3}$	$2{,}3\cdot10^{-3}$	$2{,}3\cdot10^{-3}$
Curve 2	$5{,}7\cdot10^{-12}$	$3{,}8\cdot10^{-42}$	$3{,}8\cdot10^{-42}$	$3{,}8\cdot10^{-42}$	$3{,}8\cdot10^{-42}$	$3{,}8\cdot10^{-42}$
Curve 3	0,5	$7{,}5\cdot10^{-13}$	$1\cdot10^{-45}$	Less than $1\cdot10^{-45}$	Less than $1\cdot10^{-45}$	Less than $1\cdot10^{-45}$
Curve 4	1	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$
Signal-to-noise ratio	0,01	0,1	0,5	1,0	10	100

To visualize the dependence of the B1 modem error probability on the signal-to-noise ratio, the graphs in Figure 5.6 are presented. The figure shows that curves 1,2,3 differ significantly from each other. This means that $e_ш$ - the expectation of "non-white" Gaussian noise has a strong influence on the noise immunity of the considered modem B1. The initial state of noise immunity is characterized by curve 3, obtained when modem B1 operates in a channel with "white" noise, when $e_ш = 0$. And it is a completely different matter when the product $\rho \times e_ш = 0{,}08$, which is calculated with a coefficient ρ =0,1. In this case, the noise immunity of modem B1 in the channel with "non-white" Gaussian noise (curve 2) increases by almost 10 dB compared to its noise immunity (curve 3) in the channel with "white" noise (if the comparison is performed at the error probability level of $P = 10^{-12}$).

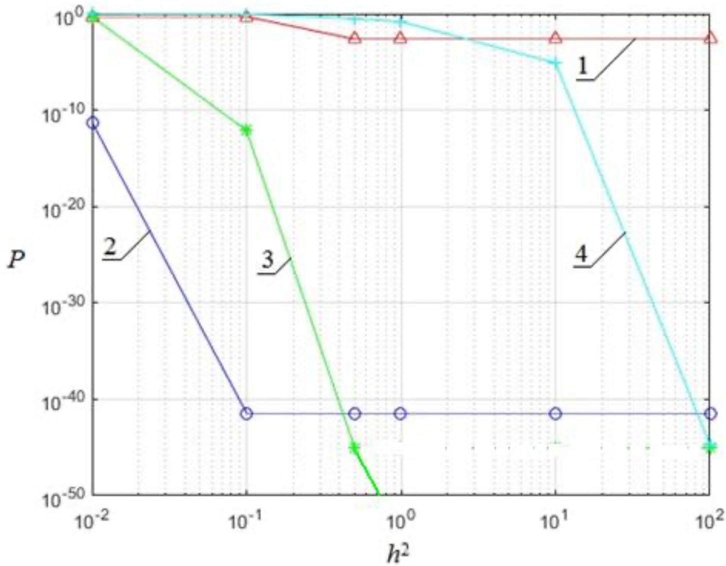


Figure 5.6. *B1 modem error probability in a channel*

with "non-white" Gaussian noise

Indeed, in this case, curve 2 then $h^2 = 0{,}1$ rises by three orders of magnitude, and the error probability decreases to the value $P = 10^{-42}$. The probability $P = 10^{-42}$ (curve 2) is greater than the probability $P = 10^{-45}$ (curve 3), however, at this stage of research, these two probabilities are equivalent for us. It can even be considered that a small expectation of Gaussian noise ($e_ш \leq 0{,}1$) has a positive effect on the operation of the B1 modem.

The situation changes with the value of the product $\rho \times e_ш = 0{,}4$. Curve 3 rises sharply and is located parallel to the x-axis at the probability level $P = 10^{-3}$ (curve 1). At the same time, it is not necessary to talk about the maximum noise immunity of the B1 modem. Here, additional measures are needed to compensate $e_ш$. Recommendations for eliminating the influence $e_ш$ on the noise immunity of the modem were formulated earlier in section 5.1.1 and recorded in [37].

As a result, the behavior of modem B1 in a channel with "non-white" Gaussian noise is ambiguous. Its noise immunity first increases when the expectation of the Gaussian noise does not exceed 0.1, and then drops noticeably if the expectation of the noise reaches 0.2 and continues to grow. At a coefficien ρ = 1 and value $e_ш$ = 0,4 we get curve 1. And this is not the end. Further increase in the expectation of noise to a value of 0.8 straightens curve 1 to a straight line running parallel to the x-axis at the error probability level P = 0,5. Again, the expectation of Gaussian noise affects the noise immunity of modem B1 only in a wired communication channel. In the radio channel, the antenna-feeder system at the input of the receiver filters the expectation of noise and thereby eliminates its effect on the noise immunity of modem B1.

Comparison of the noise immunity of modem B1 with the same characteristic of a known device in which ideal PM is used (curve 4) shows its superiority by at least 30 dB with an error probability $P = 10^{-12}$, if the value is $e_ш \leq 0{,}1$. At a value $e_ш > 0{,}2$ superiority disappears.

5.3.1. Noise immunity of the modem T2 when accepting an additive mixture of Gaussian noise and a signal with the distribution of instantaneous values according to Tikhonov law with the parameter *D* = 5

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its cipher was recorded earlier as a **T2 modem**. Let us repeat the analysis of the noise immunity of the T2 modem in the channel with "non-white" Gaussian noise at the value of the Tikhonov distribution parameter D =5. The modulation algorithm for a quasi-deterministic signal (2.33) in its previous form is written in Table 5.23. The method and results of modem research are published in [34].

Table 5.23.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,228	0
logical "1"	0,228	0,8

Let us turn to the analysis of the noise immunity of the T2 modem under the action of an additive mixture of a quasi-deterministic signal (2.33) and "non-white" Gaussian noise with expectation at its input

$$z(t)=u(t)+n(t), \tag{5.23}$$

where *n(t)* – the Gaussian noise, *u(t)* – signal (2.33).

Using expressions (2.26,2.27) and the data in Table 5.23, using formulas (3.13), we calculate the thresholds in the demodulator. As a result, at the value V_m = 1 and D = 5 we get

$$\varPi_1 = \frac{I_1(D)}{I_0(D)}\sin(e_0) = 0{,}64, \quad \varPi_2 = \frac{I_1(D)}{I_0(D)} = 0{,}9.$$

At the value V_m=1, we define for the additive mixture (5.23) the real part of the ch.f.

$$A(1,t) = \int_{-\infty}^{\infty}\cos(z)W(z)dz = \frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(\rho e_{ш}), \tag{5.24}$$

where $h=\sigma_c/\sigma_ш$ - signal-to-noise ratio: $e_ш$ – expectation of Gaussian noise; ρ – is the coefficient. When *s(t)*=0, similarly to (5.24) we calculate at the value V_m=1 for the additive mixture (5.23) the imaginary part of the ch.f.

$$B(1,t) = \int_{-\infty}^{\infty}\sin(z)W(z)dz = \frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin(\rho e_{ш}). \tag{5.25}$$

The results (5.24), (5.25) require a quantitative analysis. Tables 5.24, 5.25 present the results of calculations at $\varPi_1 = 0{,}64$, $\varPi_2 = 0{,}9$, $K_1 = 0{,}635$, $K_2 = 0{,}78$, $e_0 = 0{,}8$, $e_ш = 0{,}8$, written in a line with the name of the evaluation. In addition, in tables 5.24, 5.25, the values of the coefficient ρ are recorded in a separate column on the right.

Table 5.24.

The values of the estimate of the ch.f. in the cosine channel of the modem

Threshold $\varPi_{2k}$	0,9·0,78 = 0,7						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	0,202	0,564	0,62	0,627	1

Evaluation $\hat{A}(1,t)$	0	0	0,267	0,746	0,82	0,829	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,289	0,807	0,887	0,897	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.25.

The values of the evaluation of the ch.f. in the sinus channel of the modem

Threshold Π_{1c}	0,64·0,625 = 0,4						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0	0	0,208	0,581	0,638	0,646	1
Evaluation $\hat{B}(1,t)$	0	0	0,113	0,315	0,347	0,35	0,5
Evaluation $\hat{B}(1,t)$	0	0	0,023	0,065	0,071	0,072	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Analysis of the data in Table 5.24 shows that in the cosine channel of the T2 modem, the logical "1" is determined with errors, the probability of which depends on $e_ш$ – the mathematical expectation of "non-white" Gaussian noise. When $e_ш$=0,8 (ρ =1) there will be continuous errors in the cosine channel. A similar conclusion regarding errors when accepting a logical "0" in the sinus channel of the T2 modem follows after analyzing the data in Table 5.25.

Let the additive mixture (5.23) contain a non-centered quasi-deterministic signal at the demodulator input, this corresponds to the condition *s(t)*=1. Similarly to (5.24) at the value V_m=1 we get

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z-e_0)dz=\frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(e_0+\rho e_ш) \quad (5.26)$$

or similarly (5.25) at the value V_m=1 we calculate

$$B(1,t)=\int_{-\infty}^{\infty}\sin(z)W(z-e_0)dz=\frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin(e_0+\rho e_ш)\,. \quad (5.27)$$

Table 5.26.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	0,9·0,78 = 0,7						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	-0,008	-0,024	-0,026	-0,026	1
Evaluation $\hat{A}(1,t)$	0	0	0,105	0,294	0,323	0,326	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,185	0,516	0,567	0,573	0,1

Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.27.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{1c}	0,64·0,625 = 0,4						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0	0	0,29	0,81	0,87	0,9	1
Evaluation $\hat{B}(1,t)$	0	0	0,27	0,755	0,83	0,84	0,5
Evaluation $\hat{B}(1,t)$	0	0	0,224	0,624	0,686	0,694	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

The results (5.26), (5.27) require a quantitative analysis. Tables 5.26, 5.27 show the calculation data at $\Pi_1 = 0{,}64$, $\Pi_2 = 0{,}9$, $K_1 = 0{,}625$, $K_2 = 0{,}78$, $e_0 = 0{,}8$, $e_ш = 0{,}8$, written in a line with the name of the evaluation. In addition, in tables 5.26, 5.27, the values of the coefficient ρ are recorded in a separate column on the right.

Table 5.26 presents the results of an ideal discrimination of a logical "0" in the cosine channel of the modem T2 in the signal-to-noise ratio range of 50 dB, for which the lower limit is minus 30 dB. There are no errors here within the values $0{,}08 \le e_ш \le 0{,}8$ of the expectation of "non-white" Gaussian noise. The data in Table 5.27 shows that a logical "1" in the sinus channel of the modem T2 is determined without errors, regardless of the value of $e_ш$ in the 20 dB signal-to-noise ratio range, which has a lower limit of zero decibels.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the modem T2. Let's introduce the following designations: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ total probability of device errors.

Quantitative assessment of the noise immunity of the modem T2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 5.24 - 5.27. Both estimates are random variables with their own properties and distribution laws. Repeating verbatim the justification and the method for calculating errors in the channels of the modem T2, set out in Section 4.1.1, we obtain the data recorded in Table 5.28.

For clarity of presentation of the data in Table 5.28, Figure 5.7 shows the dependencies of the modem T2 error probability on the signal-to-noise ratio in a channel with “non-white” Gaussian noise. Curves 1-4 refer to the sine channel of the modem T2, curves 5-8 refer to the cosine channel of the modem T2. For comparison, in the same place from [15, p.473], the error probability of a device for receiving signals with ideal PM is given (curve 9). Curves 1,5 coincide for any value of the signal-to-noise ratio.

In Figure 5.7, the graphs are very densely focused on the area $10^{-1} \le h^2 \le 10^2$. Curves 7 and 8 show good noise immunity of the cosine channel of the modem T2 in a channel with Gaussian noise. Indeed, curves 7,8 diverge at the point $h^2 = 1$, and then curve 7 goes parallel to the abscissa at the level of error probability $P = 1 \cdot 10^{-45}$, and curve 8 rises. It turns out that a small value of the expectation $(e_ш \le 0{,}1)$ of Gaussian noise increases the noise immunity of the cosine channel of the modem T2 by 20 dB at the level of error probability $P = 2 \cdot 10^{-23}$. The same is observed in the sinus channel of modem T2 (curves 3 and 4).

Table 5.28.

Probability of errors of different modems

P	0,5	0,5	0,4	0,5	0,5	0,8	Curve 1
P	0,5	0,5	$3 \cdot 10^{-5}$	$6{,}5 \cdot 10^{-3}$	$9 \cdot 10^{-3}$	0,4	Curve 2
P	0,5	0,5	$2{,}5 \cdot 10^{-26}$	$1{,}9 \cdot 10^{-41}$	$5{,}5 \cdot 10^{-44}$	0,08	Curve 3
P	0,5	0,5	$1{,}1 \cdot 10^{-17}$	$5 \cdot 10^{-28}$	$4 \cdot 10^{-32}$	0	Curve 4
P	0,5	0,5	0,5	0,5	0,5	0,8	Curve 5
P	0,5	0,5	$3{,}9 \cdot 10^{-11}$	$1 \cdot 10^{-45}$	Less than $1 \cdot 10^{-45}$	0,4	Curve 6
P	0,5	0,5	$1 \cdot 10^{-45}$	$1 \cdot 10^{-45}$	Less than $1 \cdot 10^{-45}$	0,08	Curve 7
P	0,5	0,5	$1 \cdot 10^{-45}$	$5 \cdot 10^{-28}$	$2{,}1 \cdot 10^{-23}$	0	Curve 8
P	0,9	$3{,}2 \cdot 10^{-1}$	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$	0	Curve 9
h^2	0,01	0,1	1,0	10	100	$e_ш$	

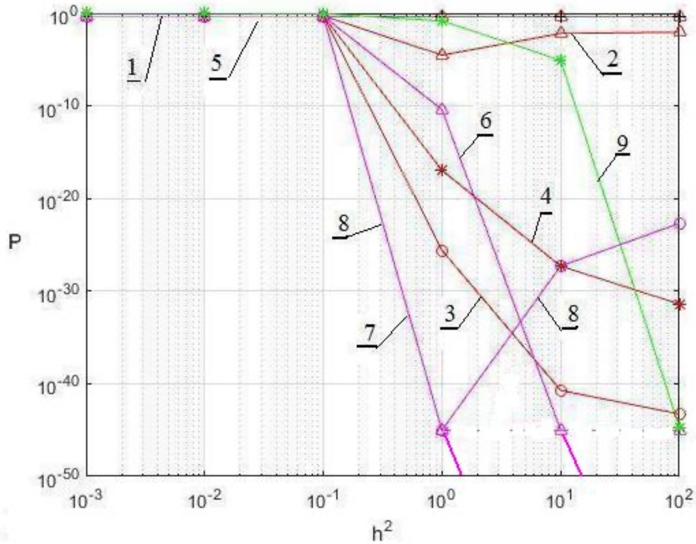


Figure 5.7. *Probability of modem T2 errors in a channel with "non-white" Gaussian noise*

Comparison of the noise immunity of the modem T2 with the noise immunity of the known device (curve 9), in which the ideal PM is used, shows the superiority of its characteristics by at least ten orders of magnitude. The cosine and sine channels of the modem T2 behave identically in the channel with Gaussian noise and show high noise immunity. To do this, it is necessary to control the expectation of "non-white" Gaussian noise in the communication channel. The necessary recommendations have already been developed and are presented in section 5.1.1. As a result, such a model of the T2 modem is promising and occupies a worthy place in the class of new generation modems.

Single-channel modem T2–1

Let the new T2-1 modem contain a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). The modulation algorithm for a quasi-deterministic signal (2.33) remains the same and is recorded in Table 5.23. At the same time, the above analysis of the noise immunity of the modem T2 in the channel with "non-white" Gaussian noise remains unchanged for the new modem model. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of modem channel transitions to the states recorded in the truth table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator shown in Figure 3.7.

Table 5.25 shows that in the sinus channel of the modem T2, the logical "0" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB, if the coefficient is $0{,}1 \le \rho \le 0{,}5$. Table 5.26 shows that in the cosine channel of the modem T2, the logical "1" is determined without errors also in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, in practice this does not work out, which is confirmed by truth table 3.1. The probability of errors in the T2-1 modem decreases on average by a factor of 20 compared to the error probability of the cosine channel of the modem T2.

The error probability of the modem T2-1is shown in Figure 5.8, where curve 1 is calculated for the value $e_{ш}$ =0,8; curve 4 - for the value $e_{ш}$ =0,4; curve 3 - for the value $e_{ш}$ =0,08; curve 2 - for the value $e_{ш}$ =0. For comparison, in the same place from [15, p.473], the error probability of a device for receiving signals with ideal PM is given (curve 5). The main values of the modem T2-1 error probability are recorded in Table 5.29.

Table 5.29.

Probability of errors of different modems

P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	0,8	Curve 1
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$1{,}9\cdot10^{-12}$	$5\cdot10^{-47}$	Less than $1\cdot10^{-47}$	0,4	Curve 2
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$5\cdot10^{-47}$	Less than $1\cdot10^{-47}$	Less than $1\cdot10^{-47}$	0,08	Curve 3
P	0,5	0,5	$1\cdot10^{-45}$	$5\cdot10^{-28}$	$2{,}1\cdot10^{-23}$	0	Curve 4
P	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$	0	Curve 5
h^2	0,01	0,1	1,0	10	100	$e_{ш}$	

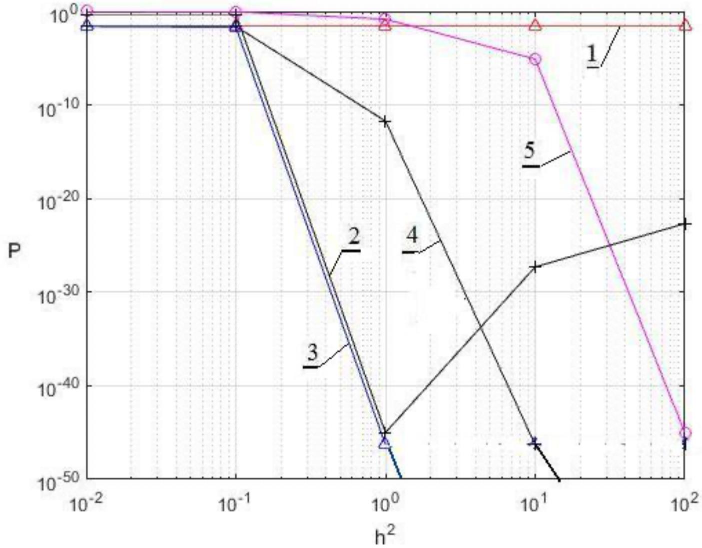


Figure 5.8. *Probability of modem T2-1 errors in a channel with "non-white" Gaussian noise*

An analysis of the graphs in Fig. 5.8 shows that the modem T2-1 works fine in a channel with "non-white" Gaussian noise. It has an error probability of $1{\cdot}10^{-47}$ in the range of signal-to-noise ratios from 10^{-1} to 10^{1}, i.e. 20 dB with a lower limit of minus 10 dB if the value $e_{ш}$ = 0,08. When the value $e_{ш}$ = 0, then the error probability of the modem T2-1starts to increase to the value of $1{\cdot}10^{-23}$ if the signal grows. It turns out that small values of the expectation of "non-white" Gaussian noise increase the noise immunity of the modem T2-1. Therefore, compensation algorithms (5.6), (5.7) for the expectation of Gaussian noise in the modem T2–1 should be reduced to the form

$$\widehat{A}(V_m) = \frac{1}{N}\sum_{k=1}^{N}\cos\left[V_m\left[z(k\Delta t) + 0{,}1 - \widehat{m}_1\{n(t)\}\right]\right], \tag{5.28}$$

$$\widehat{B}(V_m) = \frac{1}{N}\sum_{k=1}^{N}\sin\left[V_m\left[z(k\Delta t) + 0{,}1 - \widehat{m}_1\{n(t)\}\right]\right]. \tag{5.29}$$

And then the potential noise immunity of the modem T2-1 will become limiting, and the error probability will lie at the level of $1{\cdot}10^{-47}$ in the range of signal-to-noise ratios of 20 dB, in which the lower limit is equal to minus 10 dB. In this range, the modem T2-1 is superior in noise immunity to a device for receiving signals with ideal PM (curve 5).

The T2-1 modem has no analogues and competitors all over the world. It can handle signals with 10 times less noise power.

5.3.2. Noise immunity of the modem T1 when accepting an additive mixture of Gaussian noise and a signal with the distribution of instantaneous values according to Tikhonov centered law

The modem contains a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.9). Its cipher was recorded earlier as a **modem T1**. The modulation algorithm of a quasi-deterministic signal (2.33) is repeated in Table 5.30

Table 5.30.

Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the distribution parameter of the Tikhonov law
logical "0"	1,604	1
logical "1"	0,228	5

The research methodology and results are published in [32 – 34,38].

Let's proceed to the analysis of the noise immunity of the modem T1. Suppose an additive mixture of a centered quasi-deterministic signal (dynamic chaos with Tikhonov's law) and "non-white" Gaussian noise act at the demodulator input (Fig. 3.9)

$$z(t)=u(t)+n(t), \tag{5.30}$$

where *u(t)* – a signal (2.33) with the distribution of instantaneous values according to Tikhonov law, *n(t)* - "non-white" Gaussian noise with a characteristic function of the form $\Theta(V_m)=\exp\left(-\frac{V_m^2\sigma_ш^2}{2}\right)\exp(jV_m e_ш)$, V_m - a parameter of the characteristic function (ch.f.); $\sigma_ш^2$ - the dispersion (average power) of the noise.

At the value V_m=1 we define for the additive mixture (5.30) the real part of the ch.f.

$$A(1,t)=\int_{-\infty}^{\infty}\cos(z)W(z)dz=\frac{I_1(D)}{I_0(D)}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(\rho e_ш), \tag{5.31}$$

where $h=\sigma_c/\sigma_ш$ - signal-to-noise ratio; ρ - coefficient; D - parameter of Tikhonov distribution law; $I_n(\cdot)$ - the Bessel function of the imaginary argument of the n-th order of the first kind. In expression (5.31), the signal dispersion σ_c^2 changes in accordance with the modulation algorithm written in table 5.30,

at $s(t)=1$ parameter D = 5, and dispersion $\sigma_c^2=\sigma_1^2=0{,}228$;

at $s(t)=0$ parameter D = 1, and dispersion $\sigma_c^2=\sigma_0^2=1{,}604$,

where $s(t)$ - a telegraph signal in the form of a sequence of logical "0" and logical

"1". The time dependence A(1,*t*) appeared due to the telegraph signal. Taking this into account, we write down the value of the ch.f.

$$A(1,t)=\frac{I_1(5)}{I_0(5)}\exp\left(-\frac{\sigma_1^2}{2h_1^2}\right)\cos(\rho e_ш) \text{ at } s(t)=1 \text{ and}$$

$$A(1,t)=\frac{I_1(1)}{I_0(1)}\exp\left(-\frac{\sigma_0^2}{2h_0^2}\right)\cos(\rho e_ш) \text{ at } s(t)=0, \qquad (5.32)$$

where $h_0 = \sigma_0 / \sigma_ш$ - the signal-to-noise ratio when accepting a logical "0";

$h_1 = \sigma_1 / \sigma_ш$ - signal-to-noise ratio when accepting a logical "1".

The results (5.32) need to be analyzed quantitatively. Tables 5.31, 5.32 present the calculation data at $П_{1c} = 0{,}75$, $e_ш = 0{,}8$, written in a line with the name of the evaluation. In addition, in tables 5.31, 5.32, the values of the coefficient ρ are recorded in a separate column on the right.

When the modem is operating in a noisy channel, it is impossible to know the different signal-to-noise ratio at its input when receiving a logical "0" and a logical "1", because the noise power in the channel does not depend on the telegraph signal. In our example, the signal dispersions during the transmission of elements of a telegraph message correlate with each other as σ_0^2/σ_1^2=7,03508. In this regard, in a noisy channel, the ratio is $h_0^2 = 7{,}03508h_1^2$ at a constant average noise power. In addition, the demodulator (Fig. 3.9) with some error measures the value of only the real part of the characteristic function, so the threshold device receives an evaluation of the ch.f. in the form $\hat{A}(1,t)$..

In Table 5.32, all evaluation values $\hat{A}(1,t)$ are less than the threshold for any signal-to-noise ratio, regardless of the coefficient ρ. This means that in a channel with "non-white" Gaussian noise, errors when accepting a logical "0" modem T1 does not have a signal-to-noise ratio of 50 dB in the range. In Table 5.31, the evaluation values $\hat{A}(1,t)$ exceed the threshold for a signal-to-noise ratio of 1 to 100 when the coefficient $0 \le \rho \le 0{,}5$. Here in the channel

Table 5.31.

The values of the evaluation of the ch.f. in the modem channel

Threshold $П_{1c}$	0,75						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	0,2	0,564	0,62	0,627	1
Evaluation $\hat{A}(1,t)$	0	0	0,267	0,746	0,82	0,829	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,289	0,807	0,887	0,897	0,1
Relation h_1^2	0,001	0,01	0,1	1,0	10	100	

Table 5.32.

The value of the evaluation of the ch.f. in the modem channel

Threshold Π_{1c}	0,75						Coefficient ρ
Evaluation $\widehat{A}(1,t)$	0	0	0,1	0,279	0,31	0,31	1
Evaluation $\widehat{A}(1,t)$	0	0	0,132	0,368	0,407	0,44	0,5
Evaluation $\widehat{A}(1,t)$	0	0	0,142	0,399	0,441	0,445	0,1
Relation h_0^2	0,007	0,07	0,703	7,035	70,35	703,5	

Gaussian noise there are no errors when accepting a logical "1" modem T1. At $h_1^2 = 1$ and the value of the coefficient $\rho = 0{,}5$ the modem has errors in the channel with Gaussian noise when accepting a logical "1". Thus, at a value $\rho = 0{,}5$ the signal-to-noise ratio range is only 10 dB. If the value is $\rho \leq 0{,}1$, then the signal-to-noise ratio range is increased to 20 dB. It turns out that the expectation of "non-white" Gaussian noise affects the noise immunity of the modem T1, and at $\rho = 1$ and value of $e_ш = 0{,}8$ and the value of the error when accepting a logical "1" is constantly present in the modem for any signal-to-noise ratio.

We will draw the final conclusions about the noise immunity of the modem T1 according to the data in Table 5.31 (the accepted designations h_0^2, h_1^2 are further simplified to the form h^2). Its analysis shows that the noise immunity of the modem will be limiting in the range of signal-to-noise ratios from 1 to 100, i.e. in the range of 20 dB, if the coefficient $\rho \leq 0{,}1$. This indicates that the expectation operator in the mathematical model of ch.f. reliably protects the signal from noise.

Let's move on from a qualitative data analysis to a quantitative assessment of the noise immunity of the modem T1. Let's introduce the following designations: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ - the total probability of device errors.

Quantitative assessment of the noise immunity of the modem T1

The demodulator (Fig. 3.9) measures the value of the estimate of the real part of the ch.f. And, as a result of this, we obtain $\widehat{A}(1,t)$ - an estimate of the real part of the ch.f. Estimated ch.f. is a random variable that has its own properties and distribution law. Repeating verbatim to the conditions of our problem the methodology for calculating errors in the demodulator, written in detail in Section 4.1.1, we obtain the data included in Table 5.33. Curve 1 was obtained at a value of $\rho = 0.5$, curve 2 - at a value of $\rho = 0.1$, curve 3 - at a value of $\rho = 0$. For comparison, in the same place from [15, p.473], the error probability of a device for receiving signals with ideal PM is given (curve 4).

To visualize the dependence of the modem T1 error probability on the signal-to-noise power ratio, the graphs in Figure 5.9 are presented. The figure shows that curves 1,2,3 differ significantly from each other. This means that $e_ш$ - the expectation of "non-white" Gaussian noise has a strong influence on the noise immunity of the modem T1 over the entire range of signal-to-noise ratios. Therefore, we select two sections in the figure, namely: the first, where $0{,}1 \le h^2 \le 10$, and the second, where $10 < h^2 \le 100$. Let's consider each section separately.

Table 5.33.

Probability of errors of different modems

Curve 1	0,5	$2{,}8 \cdot 10^{-1}$	$2{,}1 \cdot 10^{-23}$	$2{,}8 \cdot 10^{-28}$
Curve 2	0,5	$3{,}8 \cdot 10^{-16}$	$1 \cdot 10^{-45}$	$1 \cdot 10^{-45}$
Curve 3	0,5	$3{,}8 \cdot 10^{-16}$	$2 \cdot 10^{-51}$	$2 \cdot 10^{-52}$
Curve 4	0,9	$1{,}5 \cdot 10^{-1}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-45}$
Relation h^2	0,1	1	10	100

In the first section of the figure, curves 2 and 3 coincide up to the ratio $h^2 = 1$, and then diverge. Curve 3 shows the error probability of the modem T1 when operating in a channel with "white" noise, when $e_ш = 0$. In this case, the noise immunity of the modem turns out to be maximum, and it drops by 1.25 dB if the coefficient $\rho = 0{,}1$, and $e_ш = 0.8$. This result is obtained by comparing the abscissas of curves 2,3 at the level of error probability 10^{-40}. Comparing the abscissas of curves 1,3 at the error probability level $P = 10^{-13}$, we see a drop in modem noise immunity by 7 dB when the coefficient $\rho = 0{,}5$, $e_ш = 0{,}8$. Let's continue the analysis of the figure. Suppose the coefficient $\rho = 1$ and $e_ш = 0.8$. Then, regardless of the signal-to-noise ratio, curve 1 is transformed into a straight line running parallel to the x-axis at the level $P = 0{,}5$ As a result, the noise immunity of the modem T1 reaches a minimum.

In the second section of the figure, curves 1,2,3 run almost parallel to each other at different levels of error probability. It ranges from 0.5 at value $e_ш = 0{,}8$ ($\rho = 1$) to 10^{-51} at value $e_ш = 0$. This shows that the expectation of "non-white" Gaussian noise must be reduced. Recommendations on this matter were formulated earlier in section 5.1.1 and published in [37]. It should be noted that the expectation of "non-white" Gaussian noise affects the noise immunity of the modem T1 only in a wired communication channel. In the radio channel, the antenna feeder system at the receiver input filters the expectation of noise and thereby eliminates its effect on the noise immunity of the modem T1.

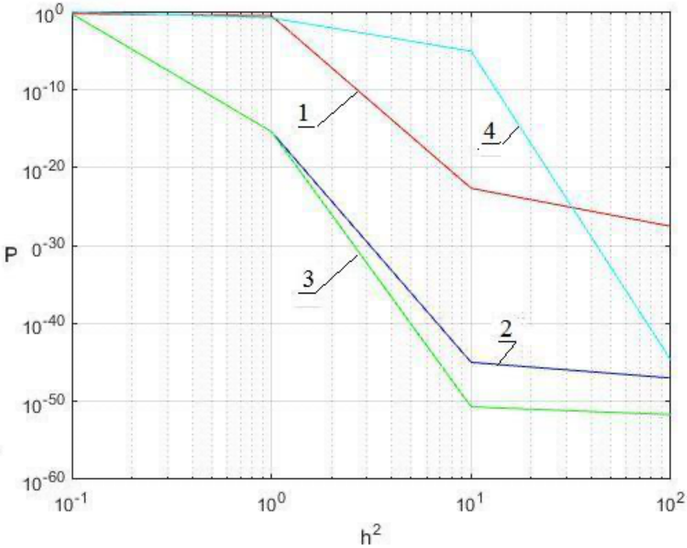


Figure 5.9. *Probability of modem T1 errors in a channel with "non-white" Gaussian noise*

Comparison of the noise immunity of the modem T1 with the noise immunity of the known device, in which the ideal PM is used (curve 4), shows the superiority of the modem T1 by at least 5 dB with the error probability $P = 10^{-22}$, if the value of $e_ш < 0,4$. At value of $e_ш \geq 0,4$ and $h^2 \geq 30$ superiority is lost.

5.4. Noise immunity of the modem K2 when receiving an additive mixture of Gaussian noise and a signal with the distribution of instantaneous values according to the cosine law

The modem contains a modulator (Fig. 3.3) and a two-channel demodulator (Fig. 3.7). Its cipher was recorded earlier as the **modem K2**. The modulation algorithm for a quasi-deterministic signal (2.19) is repeated in Table 5.34.

Table 5.34.
Signal modulation algorithm with V_m=1

Telegraph signal	Signal dispersion value	The value of the expectation of the signal
logical "0"	0,4674	0
logical "1"	0,4674	0,8

The modem research technique was developed in [14]. Let's proceed to the analysis of the noise immunity of the K2 modem, when an additive mixture of a quasi-deterministic signal (2.19) and "non-white" Gaussian noise acts at its input

$$z(t)=u(t)+n(t), \qquad (5.33)$$

where $n(t)$ – the "non-white" Gaussian noise, $u(t)$ – signal (2.19).
Using expressions (2.26,2.27) and the data in Table. 5.34 using formulas (3.13) we calculate the thresholds in the demodulator. As a result, with the value $V_m = 1$, we get

$$\Pi_1 = \frac{\pi}{4}\sin(e_0)=0{,}5634\ , \quad \Pi_2 = \pi/4 = 0{,}7854.$$

At the value $V_m=1$, we define for the additive mixture (5.33) the real part of the ch.f.

$$A\,(1,t) = \int_{-\infty}^{\infty} \cos(z) W(z) dz = \frac{\pi}{4}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\cos(\rho e_{ш}), \qquad (5.34)$$

where $h = \sigma_c / \sigma_ш$ the signal-to-noise ratio; $e_ш$ expectation of Gaussian noise; ρ – the coefficient. When $s(t)$=0, similarly to (5.34) we calculate at the value V_m=1 for the additive mixture (5.33) the imaginary part of the ch.f.

$$B(1,t) = \int_{-\infty}^{\infty} \sin(z) W(z) dz = \frac{\pi}{4}\exp\left(-\frac{\sigma_c^2}{2h^2}\right)\sin(\rho e_{ш}). \qquad (5.35)$$

The results (5.34), (5.35) require a quantitative analysis. Tables 5.35, 5.36 present the results of calculations at $\Pi_1 = 0{,}5634$, $\Pi_2 = 0{,}7854$, $K_1 = 0{,}53$, $K_2 = 0{,}764$, $e_ш = 0{,}9$, written in a line with the name of the evaluation. In addition, in tables 5.35, 5.36, the values of the coefficient ρ are recorded in a separate column on the right.

An analysis of the data in Table 5.35 shows that in the cosine channel of the modem K2, the logical "1" is determined without errors in the range of signal-to-noise power ratios from 1 to 100 or from 0 dB to 20 dB with a coefficient value of $\rho \le 0{,}1$. When $0{,}1 \le \rho \le 0{,}5$, then the range of signal-to-noise ratios in the cosine channel of the K2 modem narrows to 10 dB. And for the values of the coefficient $0{,}5 \le \rho \le 1$ in the cosine channel of the K2 modem, there will be continuous errors for any signal-to-noise ratio.

Table 5.35.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	0,7854 · 0,764 =0,6						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	0,05	0,388	0,48	0,49	1
Evaluation $\hat{A}(1,t)$	0	0	0,072	0,56	0,69	0,71	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,079	0,622	0,767	0,787	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

In table 5.36, logical "0" in the sinus channel of modem K2 is determined without errors, i.e. with ultimate noise immunity, with signal-to-noise power ratio from 10^{-3} to 10^{2}, i.e. in the range of 50 dB, starting from minus 30 dB, if the coefficient

Table 5.36.

The values of the evaluation of the ch.f. in the sinus channel of the modem

Threshold $П_{1c}$	0,5634 · 0,53 = 0,3						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0	0	0,063	0,49	0,603	0,62	1
Evaluation $\hat{B}(1,t)$	0	0	0,035	0,27	0,335	0,344	0,5
Evaluation $\hat{B}(1,t)$	0	0	0,007	0,056	0,069	0,071	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

This allows us to say that simple control commands such as turn on-off, open-close and others will be accepted with a certainty equal to one, in any operating conditions of the K2 modem. Then things get worse. When $0{,}1 \le \rho \le 1$, then the range of signal-to-noise ratios in the sinus channel of modem K2 is reduced to 30 dB with a lower limit equal to minus 30 dB. Here, in the sinus channel of the modem K2, the logical "0" will be determined without errors, and there will be errors outside the specified range.

Suppose the additive mixture (5.33) contain a non-centered quasi-deterministic signal at the demodulator input, this corresponds to the condition *s(t)*=1. Similarly to (5.34) at the value V_m=1 let's define

$$A(1,t) = \int_{-\infty}^{\infty} \cos(z) W(z) dz = \frac{\pi}{4} \exp\left(-\frac{\sigma_c^2}{2h^2} \right) \cos(e_0 + \rho e_{ш}) \qquad (5.36)$$

or similarly (5.35) at the value V_m=1 let's calculate

$$B(1,t) = \int_{-\infty}^{\infty} \sin(z) W(z) dz = \frac{\pi}{4} \exp\left(-\frac{\sigma_c^2}{2h^2} \right) \sin(e_0 + \rho e_{ш}) . \qquad (5.37)$$

The results (5.36), (5.37) require a quantitative analysis. Tables 5.37, 5.38 show the calculation data at $П_1 = 0{,}5634$, $П_2 = 0{,}7854$, $K_1 = 0{,}53$, $K_2 = 0{,}764$, $e_{ш} = 0{,}9$, written in a line with the name of the evaluation. In addition, in tables 5.37, 5.38, the values of the coefficient ρ are recorded in a separate column on the right.

At the selected threshold values according to Table 5.38, the discrimination of logical "1" from zero in the sinus channel of modem K2 occurs without errors in the range of signal-to-noise ratios from 1 to 100, i.e. in the range, equal to 20 dB, at any value of the coefficient ρ. At the same time, in the cosine channel of the K2 modem (Table 5.37), when determining the logical "0", the maximum noise immunity is maintained at a signal-to-noise ratio of 10^{-3} to 10^{2}, i.e. in the range of 50 dB, in which the lower limit is equal to minus 30 dB, and it does not depend on the value of the coefficient ρ. Therefore, simple control commands such as turn on-off, close-open and others will be accepted by the cosine channel with a reliability equal to one, under any operating conditions of the modem K2.

Table 5.37.

The values of the evaluation of the ch.f. in the cosine channel of the modem

Threshold Π_{2k}	$0{,}7854 \cdot 0{,}764 = 0{,}6$						Coefficient ρ
Evaluation $\hat{A}(1,t)$	0	0	- 0,01	-0,08	- 0,1	- 0,1	1
Evaluation $\hat{A}(1,t)$	0	0	0,025	0,197	0,243	0,25	0,5
Evaluation $\hat{A}(1,t)$	0	0	0,05	0,393	0,485	0,497	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

Table 5.38.

The values of the evaluation of the ch.f. in the sinus channel of the modem

Threshold Π_{1c}	$0{,}5634 \cdot 0{,}53 = 0{,}3$						Coefficient ρ
Evaluation $\hat{B}(1,t)$	0	0	0,079	0,619	0,764	0,783	1
Evaluation $\hat{B}(1,t)$	0	0	0,076	0,592	0,73	0,75	0,5
Evaluation $\hat{B}(1,t)$	0	0	0,062	0,481	0,594	0,61	0,1
Relation h^2	0,001	0,01	0,1	1,0	10	100	

As a result of the analysis of the noise immunity of the modem K2, we can say that in the presence of "non-white" Gaussian noise in the data transmission channel, the Kotelnikov noise immunity of the modem K2 depends on the value of the expectation of noise. At accurate synchronization of the operation of both channels of the K2 modem, there are no errors when accepting a telegraph signal in the range of signal-to-noise ratios of 20 dB or more, with the lower limit of the range equal to 0 dB, and the value of the expectation of Gaussian noise $e_{ш} \leq 0{,}1$.

Let's move from qualitative data analysis to quantification noise immunity modem K2. Let's take the following designations: P_0 – the probability of errors when accepting a logical "0"; P_1 – the probability of errors when accepting a logical "1"; $P = \frac{1}{2}(P_0 + P_1)$ the total probability of device errors.

Quantitative assessment of the noise immunity of the modem K2

In expressions (3.11,3.12), instead of the expectation operator, an ideal adder is used. And, as a result of this, we obtain estimates of the real and imaginary parts of the ch.f., which are recorded in tables 5.35 - 5.38. Both evaluations are random variables with their own properties and distribution laws. Repeating verbatim to the conditions of our problem the method of calculating errors in the demodulator, written in detail in Section 4.1.1, we obtain the data included in Table 5.39.

To visualize the error probability of the K2 modem depending on the signal-to-noise ratio and the value of ρ, the graphs in Figure 5.5 are presented. Curves 1 - 4 characterize the error probability of the sine channel, and curves 5 - 8 characterize the cosine channel of the K2 modem. Curve 9 shows the error probability of the device for receiving signals with ideal PM according to the work [15, p.473]. On fig. 5.10 curves 1.5 coincide at any signal-to-noise ratio, and curves 7,8 coincide in the section $10 \le h^2 \le 100$ and in the section $10^{-3} \le h^2 \le 10^{-1}$. In addition, in the section $0{,}1 \le h^2 \le 10$ curves 7 and 8 are so close to each other, that it is sometimes difficult to distinguish them. This means that the expectation of Gaussian noise at value of $e_ш \le 0{,}1$ has almost no effect on the noise immunity of the cosine channel of the K2 modem, but it has a positive effect on the operation of the sine channel. In the sinus channel of modem K2 (curves 3.4), noise immunity increases by 4.26 dB. This means that the signal modulation algorithm in Table. 5.34 is not optimal and can be adjusted.

Table 5.39.

Probability of errors of different modems

P	0,5	0,5	0,5	0,5	0,5	0,9	Curve 1
P	0,5	0,5	$8·10^{-2}$	$5·10^{-2}$	$1{,}9·10^{-2}$	0,45	Curve 2
P	0,5	0,5	$1{,}1·10^{-17}$	$5{,}5·10^{-44}$	$1·10^{-45}$	0,09	Curve 3
P	0,5	0,5	$5{,}5·10^{-13}$	$4·10^{-32}$	$4{,}3·10^{-35}$	0	Curve 4
P	0,5	0,5	0,5	0,5	0,5	0,9	Curve 5
P	0,5	0,5	0,5	$2{,}1·10^{-37}$	$1·10^{-45}$	0,45	Curve 6
P	0,5	0,5	$1{,}1·10^{-5}$	$1·10^{-45}$	Less than $1·10^{-45}$	0,09	Curve 7
P	0,5	0,5	$9{,}3·10^{-4}$	$1·10^{-45}$	Less than $1·10^{-45}$	0	Curve 8
P	0,9	$3{,}2·10^{-1}$	$1{,}5·10^{-1}$	$8·10^{-6}$	$2·10^{-45}$	0	Curve 9
h^2	0,01	0,1	1,0	10	100	$e_ш$	

Apparently, it would be more correct to write $e_0 = 0{,}9$. Then the probability of errors in the sinus channel of the K2 modem at $h^2 = 1$ will decrease by four orders of magnitude up to the value $P = 1{,}1·10^{-17}$. Moreover, in the cosine channel of the

K2 modem, the probability of errors will decrease only by one order and will be $P = 1{,}1 \cdot 10^{-5}$ (table 5.39).

And it's a completely different matter when the value of $e_{ш} > 0{,}1$. When $e_{ш} = 0{,}9$ ($\rho = 1$) in both modem channels, the error probability is 0.5 (curves 1.5) for any signal-to-noise ratio. Here, the noise immunity of the modem K2 in the channel with Gaussian noise reaches a minimum, as a result of which it becomes inoperable. To ensure the operation of the K2 modem with high noise immunity in a channel with Gaussian noise, additional measures are required. The content of these activities is described in Section 5.1.1.

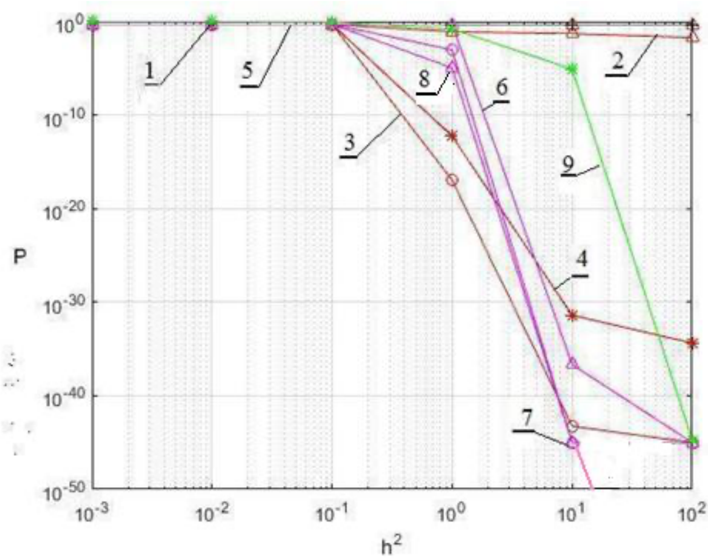


Figure 5.10. *Error probability of modem K2 in a channel with "non-white" Gaussian noise*

Comparison of the noise immunity (curves 3,6,7,8) of the K2 modem with the noise immunity (curve 9) of a known device in which ideal PM is used shows its superiority by thirteen orders and up to thirty orders in a channel with "non-white" Gaussian noise. The noise immunity of the K2 modem in the channel with "non-white" Gaussian noise in the section $10 \le h^2 \le 100$ is even better than in the channel with "white" noise. This follows from the comparison of curve 4 with curves 3,6,7,8.

Single-channel modem K2 -1

Let the K2 modem contain a modulator (Fig. 3.3) and a single-channel demodulator (Fig. 3.8). The modulation algorithm for a quasi-deterministic signal (2.19) is written in Table 5.34. At the same time, the above analysis of the noise immunity of the modem K2 when operating in a channel with "non-white" Gaussian noise remains unchanged for the new model of the K2 modem - 1. However, the new modem has only one channel and one output, on which the telegraph signal will appear as a result of the transition of the modem channels to the states recorded in the truth table 3.1. Let's recall that the demodulator (Figure 3.8) combines the advantages of the sine and cosine channels of the demodulator shown in Figure 3.7.

Table 5.36 shows that in the sinus channel of the demodulator, a logical "0" is determined without errors in the range of signal-to-noise ratios from 10^{-3} to 10^{-1}, i.e. in the range of 30 dB. Table 5.37 shows that in the cosine channel of the demodulator, the logical "1" is determined without errors in the entire range of signal-to-noise power ratios, i.e. in the range of 50 dB. When combining these advantages of both channels together, we get a new modem with maximum noise immunity in the range of signal-to-noise ratios of 50 dB, with the lower limit of the range equal to minus 30 dB. However, in practice this does not work out, which is confirmed by truth table 3.1. The probability of errors in the modem K2-1 is reduced by an average of 20 times compared with the probability of errors in the cosine channel of the modem K2.

The error probability of the modem K2-1 is presented in Table 5.40 and in Figure 5.11, where curve 1 is plotted at $e_ш = 0{,}9$(ρ =1); curve 2 – $e_ш = 0{,}45$ (ρ=0,5); curve 3 – $e_ш = 0{,}09$ (ρ =0,1); curve 4 - $e_ш = 0$ (ρ =0); curve 5 refers to a device in which phase modulation is applied. Curves 3 and 4 coincide in the section $10^{-2} \le h^2 \le 10^1$.

An analysis of the graphs in Figure 5.11 shows that the K2-1 modem works well with large signals. It has an error probability of $1 \cdot 10^{-45}$ in the range of signal-to-noise ratios from 1 to 100, and the probability of errors depends on the value of the expectation of "non-white" Gaussian noise. For example, at $e_ш = 0{,}09$ the probability of modem errors K2-1 remains at the level of $1 \cdot 10^{-45}$ (curve 3) for any signal-to-noise ratio in the range of 10 dB, starting from 10 dB. This is the best indicator of the K2-1 modem. Curve 1 in the section $10^{-2} \le h^2 \le 100$ shows the low noise immunity of modem K2-1 with an error probability of $2.5 \cdot 10^{-2}$. If the expectation of “non-white” Gaussian noise is compensated to the value $e_ш = 0{,}1$, then the noise immunity of the K2-1 modem will be restored and will correspond to curve 3. Recommendations for eliminating the expectation of “non-white” Gaussian noise are recorded in section 5.1.1.

Table 5.40.

Probability of errors of different modems

P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	0,9	Curve 1
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$1\cdot10^{-38}$	$5\cdot10^{-47}$	0,45	Curve 2
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$5{,}5\cdot10^{-7}$	$5\cdot10^{-47}$	Less than $1\cdot10^{-47}$	0,09	Curve 3
P	$2{,}5\cdot10^{-2}$	$2{,}5\cdot10^{-2}$	$4{,}6\cdot10^{-5}$	$5\cdot10^{-47}$	Less than $5\cdot10^{-47}$	0	Curve 4
P	0,9	$3{,}2\cdot10^{-1}$	$1{,}5\cdot10^{-1}$	$8\cdot10^{-6}$	$2\cdot10^{-45}$	0	Curve 5
h^2	0,01	0,1	1,0	10	100	$e_ш$	

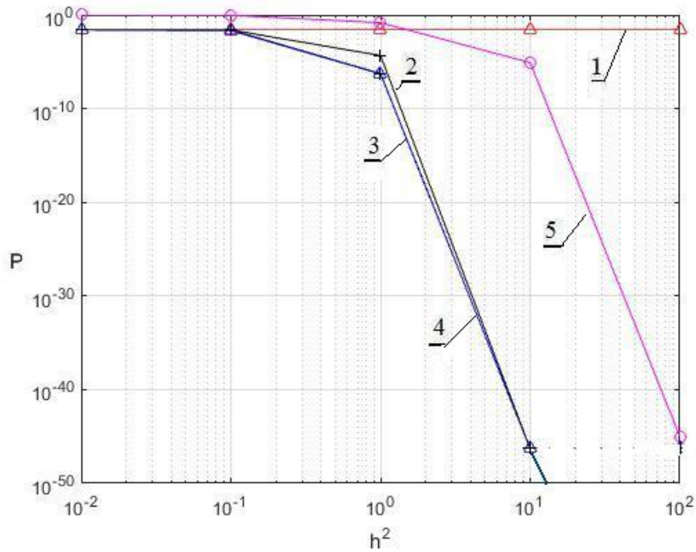


Figure 5.11. *Probability of modem K2 errors - 1 in the channel with "non-white" Gaussian noise*

6. COMPUTER SIMULATION OF THE MODEM

To confirm the results of theoretical studies, computer simulation of different models of modem A, recorded in Table 4.50, was performed. At the same time, the Matlab application software package was used, which has significant advantages over currently existing mathematical modeling programs. The Matlab package was created for scientific and engineering calculations and is focused on working with data arrays. All these features make the Matlab package very attractive for solving various problems, including modeling devices and systems for transmitting discrete information over communication channels.

The characteristics of the systems under study are entered in an interactive mode, by graphical assembly of the connection diagram of standard elementary links. The elementary links are blocks (or modules) stored in the built-in library of the Simulink environment. The composition of the library can be supplemented by the user's own developments. Any model can have a nested structure, i.e. consist of lower level models [39 – 45]. In this case, the number of nested models can be very large. Further, we will agree to call nested models subsystems.

6.1. Description of the computer model

The computer model includes a modulator (Fig. 3.1), a communication channel and a demodulator (Fig. 3.7). Together they allow us to investigate the noise immunity of all models of modem A (Table 4.50) in a channel with "white" noise.

Modeling and subsequent study of the modem was performed in the Matlab software environment. To build the model, standard modules of the Simulink base library, as well as DSP Blocksets and Communication Blocksets libraries were used. Processed signals are stored in the working space of the environment. The built-in generators of pseudo-random sequences of the Matlab package were used for noise modeling.A general view of the computer model is shown on fig. 6.1 [24]. The model requires adjustment of block parameters taken from Simulink environment libraries. In the computer model, the communication channel contains the Add1 adder block, which is used to form an additive mixture.

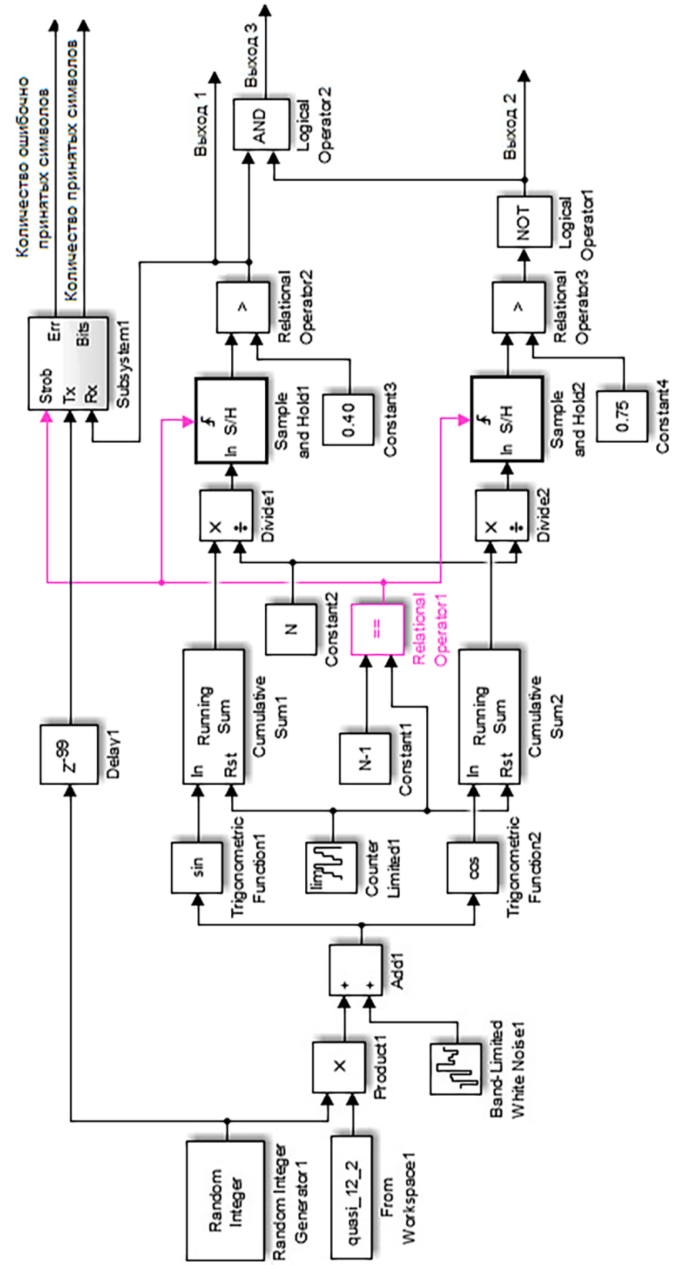


Figure 6.1. *General view of the computer model*

In the modulator, the formation of the telegraph signal *s(t)* in the form of binary parcels is performed using the Random Integer Generator1 block. This block generates a pseudo-random binary sequence that simulates the transmitted information. The Product1 block multiplies the quasi-deterministic signal *u(t)* and the pseudo-random binary sequence. To transmit one data bit, *N* samplings of a quasi-deterministic signal are used. Loading samples of a quasi-deterministic signal into the model is performed using the FromWorkspace1 block.

The demodulator is built in accordance with the block diagram in Figure 3.8, in the model its functions are extended using the Manual Switch key, which allows you to use the modem channels separately or together through a special combination scheme. Let's recall that the demodulator measures estimates of the real and imaginary parts of the Lyapunov characteristic function (ch.f.) of the additive mixture of signal and noise

$$\widehat{A}(1,t)=\frac{1}{N}\sum_{k=1}^{N}\cos\left[z(k\Delta t)\right]\ , \qquad (6.1)$$

$$\widehat{B}(1,t)=\frac{1}{N}\sum_{k=1}^{N}\sin\left[z(k\Delta t)\right]\ , \qquad (6.2)$$

where $z(t)=u(t)+n(t)$ – is an additive mixture of a quasi-deterministic signal *u(t)* and "white" noise *n(t)*; Δt is the discretization interval, *k* is the ordinal number of the discrete sample of the additive mixture. The additive mixture is fed to the input of a demodulator having sine and cosine channels. Evaluation (6.2) is measured in the sine channel, estimate (6.1) is measured in the cosine channel. The sine values of the additive mixture of the useful signal and noise are calculated using the Trigonometric Function1 block, and the cosine values are calculated using the Trigonometric Function2 block.

The sum of sines averaged over 100 values in accordance with expression (6.2) is compared with the Π_{1c} threshold set in the Constant3 block, and the sum of cosines averaged over 100 values in accordance with expression (6.1) is compared with the P_{2k} threshold set in the Constant4 block. The threshold devices are implemented using the Relational Operator2 block and the Relational Operator3 block. Each block compares the value of the average sum with a threshold. If the average sum is greater than the threshold, the threshold device outputs a logical unit. Otherwise, the threshold device outputs a logical zero. The output of the Relational Operator2 block is the output of the sinus channel (Output 1). The Relational Operator3 block is connected to a logic element "NOT" implemented using the Logical Operator1 block, whose output corresponds to the output of the cosine channel (Output 2). The threshold devices of the sine and cosine channels are combined using a special circuit. It consists of blocks Logical Operator1 ("NOT" logic element) and Logical Operator2 ("AND" logic element). Thus, after combining the sine and cosine channels, the modem signal is generated at the output of the Logical Operator2 block (Output 3). A more detailed description of the computer model in Figure 6.1 is available on the website [46].

6.2. Characteristics of a quasi-deterministic signal

Samplings of the quasi-deterministic signal are loaded into the model from the workspace of the Matlab package, into which they are previously entered from the output of the ADC connected to the source AKIP - 3409/4 of physical processes. The From Workspace1 block is used to load samplings of a quasi-deterministic signal into the model. The settings window of the From Workspace1 block contains the name of the workspace variable of the Matlab package (the Data field), as well as the values of the sampling period of the quadri-deterministic signal (the Sample time field). The signal oscillation frequency is set to 50 kHz, and the sampling period is set to $\Delta t = 2 \cdot 10^{-6}$ c.

$$\Delta t = 2 \cdot 10^{-6} \text{ s.}$$

Verification of the probabilistic characteristics of the quasi-deterministic signal of the generator AKIP - 3409/4 was performed using a virtual instrument XN 31.1 *beta* [3]. Figure 6.2 shows an estimate of the probability density of a quasi-deterministic signal.

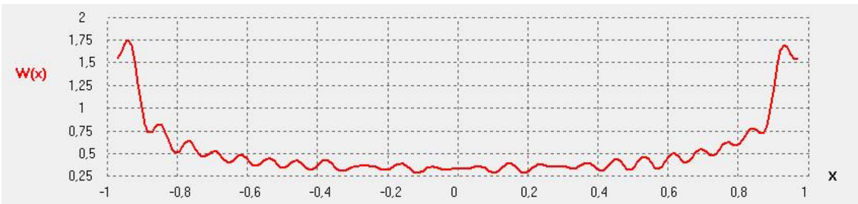


Figure 6.2. *Signal probability density evaluation*

The estimate of the probability density of the signal repeats the form of the arcsine law, is symmetrical about zero, and has maxima at values equal to the signal amplitude. Using the scale grid in Figure 6.2, we calculate the signal amplitude and get a value of 0.95. Measurements of the initial and central moment functions of the signal using the XN 31.1 *beta* device showed the following result: $\hat{m}_1 = 0{,}0067$ – an evaluation of the initial moment of the first order (the expectation of the signal); $\hat{M}_2 = 0{,}4546447$ – evaluation of the central moment of the second order (signal dispersion). Using the formula, $2\sigma_c^2 = U_0^2$ we calculate the signal amplitude, as a result of which we obtain U_0 =0,95. The scale grid in Figure 6.2 does not allow you to see the expectation of the signal with a value of 0.0067. However, its presence will adversely affect the modem simulation results.

Other evaluations of the probabilistic characteristics of a quasi-deterministic signal, measured by the XN 31.1 beta virtual instrument, do not differ from the theoretical curves obtained for a signal with a mathematical model of the form (2.1). They are not shown here, because were previously shown in Figures 2.2, 2.3, 2.4.

As a result, we can assume that the physical process of the AKIP-3409/4 source has properties similar to those of a quasi-deterministic signal (2.1) and can be used in modem modeling.

6.3. Noise characteristics

In the computer model in Figure 6.1, "white" noise is formed by the Band - Limited White Noise 1 block. The value of the dispersion of "white" noise is set in the workspace of the Matlab package, so the variable N_0 is set in the Noise Power field. The initial value of the pseudo-random number generator base (Seed field) is taken by default. The sampling period of "white" noise corresponds to the sampling period of a quasi-deterministic signal, i.e. value $\Delta t = 2 \cdot 10^{-6}$ s.

The probabilistic characteristics of the noise were investigated using the XN 31.1 *beta* virtual instrument. An evaluation of the noise probability density is presented in Figure 6.3.

The shape of the graph in Figure 6.3 repeats the form of a Gaussian curve. Therefore, we can assume that the instantaneous values of the noise are distributed according to the normal law, or, in other words, have a Gaussian distribution. The initial and central moment functions have the following value: $\hat{m}_1 = 0{,}00345 -$ evaluation of the initial moment of the first order (expectation of noise); $\hat{M}_2 = 0{,}7196696 -$ second-order central moment evaluation (noise dispersion).

Figure 6.3. *Noise probability density evaluation*

Figure 6.4 shows an estimate of the correlation function of noise, the value of which at the value $t = 0$ is equal to $\hat{M}_2 = 0{,}7$. The view of the graph in Figure 6.4 approaches the image of a delta function. When the value $t = \Delta t = 2 \cdot 10^{-6}$ of the noise correlation function is $k(t) = 0$. This means that the discrete instantaneous noise values taken over the sampling interval $\Delta t = 2 \cdot 10^{-6}$ s., are uncorrelated.

An evaluation of the noise power spectral density is shown in Figure 6.5. It has unevenness. If the density value $G(f) = 13{,}75 \cdot 10^{-2}$ is conditionally taken as average, then the unevenness of the noise power spectral density will be 1.8% up and 3.6% down. The energy bandwidth of the noise is 100 kHz. Therefore, the carrier frequency of the quasi-deterministic signal is chosen to be 50 kHz.

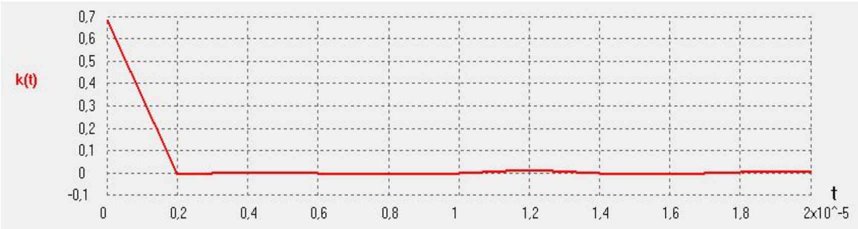


Figure 6.4. *Noise correlation function evaluation*

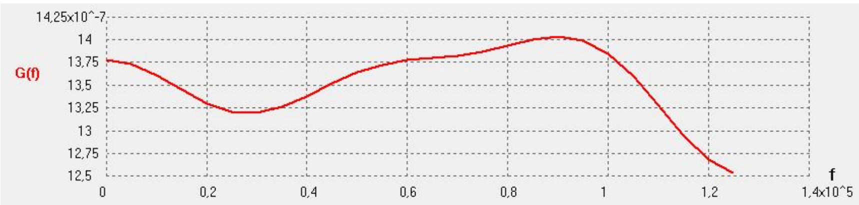


Figure 6.5. *Noise power spectral density evaluation*

At the end of the show, let's give an evaluation of the characteristic function of noise in Figure 6.6. As expected, the imaginary part of the ch.f. equals to zero. It turns out, indeed, that the expectation of noise is zero. Therefore, the previously indicated value $\hat{m}_1 = 0{,}00345$ can be considered the error of the device and nothing more.

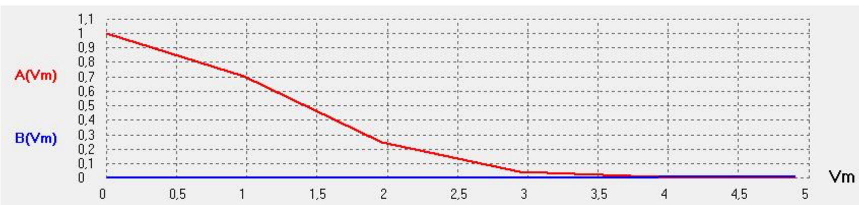


Figure 6.6. *Evaluations for the real and imaginary parts of the ch.f.*

In conclusion, based on statistical radio engineering [4], let's write down the main properties of "white" noise:

1) instantaneous noise values are distributed according to the normal law;
2) any two adjacent instantaneous noise values are uncorrelated;
3) the noise power spectral density is constant within the energy bandwidth;
4) the expectation of noise is zero.

In our opinion, the noise formed in the computer model (Fig. 6.1) satisfies all these properties, which means that it is "white".

6.4. Characteristics of estimates of the characteristic function of a quasi-deterministic signal

The modem demodulator in the absence of noise measures the values of the ch.f. evaluations of quasi-deterministic signal according to algorithms (6.1, 6.2). Ch.f. evaluations are random variables that have their own properties and distribution laws, which are taken into account when analyzing the noise immunity of the modem. Since it is theoretically possible to obtain the laws of distribution of evaluations of ch.f. difficult enough, the hypothesis was put forward that the scores are distributed according to the normal law. For the first time, this law was obtained by the empirical method based on the results of demodulator simulation using the computer model Figure 6.1 in [28] and is shown in Figure 6.7.

The cosine channel of the demodulator, which measures the estimate (6.1), is considered first. In total, one thousand evaluation values were processed $\widehat{A}(1,t)$, each of which was measured in accordance with equation (6.1) at a value of N= 100, when the amplitude of the quasi-deterministic signal (2.1) with the arcsine distribution law is U_0 = 1,5, and $n(t)=0$. The exact (principal) value of the estimate is 0.5118 and is calculated using the fundamental formula

$$A(V_m)=m_1\{\cos[V_m u\ (t)]\}=\int_{-\infty}^{\infty}\cos(V_m y)W(y)dy=J_0(V_m U_0) \quad (6.3)$$

at $V_m=1$, where $W(y)$ - the probability density (2.2) of the signal $u(t)$ with the arcsine distribution law; $J_0(\cdot)$ - Bessel function of the zeroth order of the first kind. The probability density of estimate is shown in Figure 6.7 (6.1).

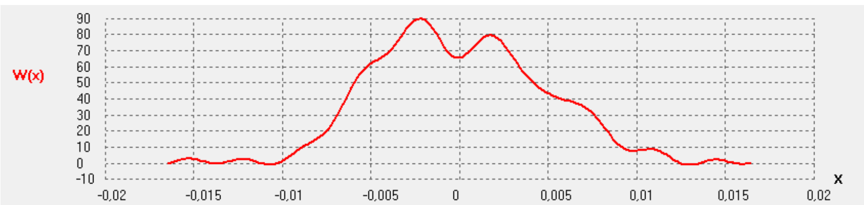


Figure 6.7. *Estimating the probability density of values* $\widehat{A}(1,t)$

The sinus channel of the demodulator, which measures estimate (6.2), is similarly studied. The evaluation probability density (6.2) repeats the graph in Figure 6.7. Therefore, we can say that the probability density of evaluations of the real and imaginary parts of the ch.f. almost repeats the Gaussian curve. This means that the law of distribution of the estimate of the real part and the evaluation of the imaginary part of the ch.f. is normal. Previously, this was discussed only hypothetically, however, with the help of experimental studies of the demodulator, the hypothesis has now been confirmed [28].

6.5. Studying of modem noise immunity using a computer model

In the computer model in Figure 6.1, white noise is generated by the Band-Limited White Noise1 block. Samples of the quasi-deterministic signal are loaded into the model from the workspace of the Matlab package, into which they are preliminarily entered with a volume of 10^7 discrete values from the output of the ADC connected to the source AKIP - 3409/1 of physical processes. The oscillation frequency of the quasi-deterministic signal is set to 50 kHz. The sampling period of the signal and "white" noise with a bandwidth of 100 kHz is $\Delta t = 2{\cdot}10^{-6}$ s. The probabilistic characteristics of both processes in statistical radio engineering are well studied, and the estimates of these characteristics were checked before the study using the XN 1.31 *beta* virtual instrument [3]. The check confirmed the status of both processes.

To determine the error probability when receiving binary messages, the computer model contains the Subsystem1 subsystem, the structure of which is shown in Figure 6.8. In this subsystem, the number of erroneously received symbols (bits) N_{err} is calculated, and the total number of received symbols N_{tot}. In the computer model in Figure 6.1, the number of transmitted and received binary characters is the same.

Subsystem1 has three inputs: Strob, Tx and Rx. The Strob input receives a control signal from the Relational Operator1 block (Fig. 6.1). The Rx input receives a binary sequence from any modem output (Output 1, Output 2, Output 3). The original binary pseudo-random sequence is fed to the Tx input through the Delay1 delay block. The Add1 block calculates the difference between the values of each transmitted bit and the corresponding received bit. The Abs1 block calculates the modulus of the difference values coming from the Add1 block. Thus, to determine the number of erroneously received characters, the original binary pseudo-random sequence is compared with the binary sequence at any output of the modem. The Cumulative Sum1 block of the Subsystem1 subsystem counts and stores the number of erroneously accepted symbols (bits).

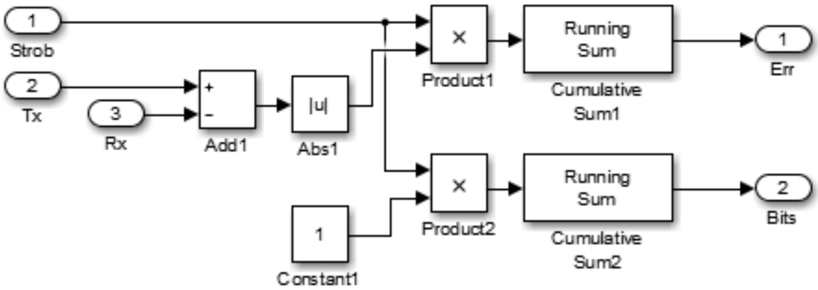


Figure 6.8. *Model subsystem for detecting errors in the transmission of binary characters*

The total number of transmitted binary symbols is counted by the single pulses received at the Strob input from the output of the Logical Operator1 block (Fig. 6.1). The total number of received symbols is counted using the Cumulative Sum2 block of Subsystem1.

The study of the noise immunity of the modem was carried out at the value N=100 in expressions (6.1), (6.2). Therefore, at the specified sampling time of the additive mixture, the duration of one symbol (bit) is equal to $N{\cdot}\Delta t = 100{\cdot}2{\cdot}10^{-6} =$ $= 2{\cdot}10^{-4}$ s. Then the information transfer rate will be 5000 bps.

Using the model in Figure 6.1, different versions of the modem (single-channel, dual-channel, with a connected channel combining scheme according to the scheme in Figure 3.8) were successively studied. The results of the modem study are recorded in tables 6.1 - 6.6, which include additional explanations and the following designations: $P_0=P_1=N_{err}/N_{tot}$ – error probability; $P_{ex}=0{,}5{\cdot}(P_0+P_1)$ – total probability of errors in modem modeling; P_{calc} – the calculated probability of modem errors; h^2 – the signal-to-noise power ratio; U_0 – amplitude and e_0 – the expectation of the quasi-deterministic signal.

In tables 6.1 - 6.5, errors are shown separately when accepting logical "0" and logical "1". Errors are different. For example, in a two-channel modem (Table 6.2) in the sinus channel, the errors are equal to zero when accepting logical "0", while in the cosine channel (Table 6.3), on the contrary, when accepting logical "1". This point allows you to get a positive effect on modem errors. When combining demodulator channels

Table 6.1.
The results of the study of a single-channel modem

Modem type: single-channel									
Suboptimal modulation algorithm: U_0=0; e_0=0 at $s(t)$=0 and U_0=1,425; e_0=0 at $s(t)$=1									
Transmission of logical "1"									
h^2		0,1	0,5	1,00	2,00	5,00	10,00	20,00	100
$П_{2к}$ = 0,6325 Cosine channel	$\boldsymbol{P_1}$	**0**	**0**	**0**	**0**	**0,00001**	**0**	**0**	**0**
	N_{err}, bites	0	0	0	0	1	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000
Transmission of logical "0"									
$П_{2к}$ = 0,6325 Cosine channel	$\boldsymbol{P_0}$	**1,0**	**1,0**	**0,71499**	**0,00584**	**0**	**0**	**0**	**0**
	N_{err}, bites	100000	100000	71499	584	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000

Table 6.2.
The results of the study of the two-channel modem

Modem type: two-channel									
Suboptimal modulation algorithm: U_0=0,594; e_0=0 at $s(t)$=0 and U_0=0,594; e_0=0,9 at $s(t)$=1									
Transmission of logical "0"									
h^2		0,01	0,10	1,00	2,00	4,00	10,00	20,00	100
$П_{1c}$ =0,4 Sinus channel	$\boldsymbol{P_0}$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	N_{err}, bites	0	0	0	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000
$П_{2к}$ = 0,75 Cosine channel	$\boldsymbol{P_0}$	**1,0**	**1,0**	**0,00001**	**0**	**0**	**0**	**0**	**0**
	N_{err}, bites	100000	100000	1	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000

Table 6.3.
The results of the study of the two-channel modem

Modem type: two-channel									
Suboptimal modulation algorithm: U_0=0,594; e_0=0 at $s(t)$=0 and U_0=0,594; e_0=0,9 at $s(t)$=1									
Transmission of logical "1"									
h^2		0,01	0,10	1,00	2,00	4,00	10,00	20,00	100
Π_{1c} =0,4 Sinus channel	$\boldsymbol{P_1}$	**1,0**	**0,09492**	**0**	**0**	**0**	**0**	**0**	**0**
	N_{err}, bites	100000	9492	0	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000
$\Pi_{2к}$ = 0,75 Cosine channel	P_1	0	0	0	0	0	0	0	0
	N_{err}, bites	0	0	0	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000

Table 6.4.
Modem study results with connected channels scheme of union

Modem type: with connected channels scheme of union									
Suboptimal modulation algorithm: U_0=0,594; e_0=0 at $s(t)$=0 and U_0=0,594; e_0=0,9 at $s(t)$=1									
Transmission of logical "1"									
h^2		0,01	0,10	1,00	2,00	4,00	10,00	20,00	100
Π_{1c} =0,15 The modem has a common output	$\boldsymbol{P_1}$	**0,98355**	**0,01074**	**0**	**0**	**0**	**0**	**0**	**0**
	N_{err}, bites	98355	1074	0	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000
Transmission of logical "0"									
$\Pi_{2к}$ = 0,67 The modem has a common output	$\boldsymbol{P_0}$	**0,01633**	**0,01549**	**0**	**0**	**0**	**0**	**0**	**0**
	N_{err}, bites	1633	1549	0	0	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000

Table 6.5.

The results of the study of a single-channel modem

Modem type: single-channel									
Optimal modulation algorithm: U_0=0; e_0=0 at $s(t)$=0 and U_0=1,1999; e_0=0 at $s(t)$=1									
Transmission of logical "1"									
h^2		0,01	0,10	1,00	1,50	4,00	10,00	20,00	100
$П_{2к}$ = 0,2 Cosine channel	$\boldsymbol{P_1}$	**1,0**	**0,00218**	**0,00208**	**0,00142**	**0,00015**	**0**	**0**	**0**
	N_{err}, bites	100000	218	208	142	15	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000
Transmission of logical "0"									
$П_{2к}$ = 0,2 Cosine channel	$\boldsymbol{P_0}$	**1,0**	**1,0**	**0,28751**	**0,00147**	**0**	**0**	**0**	0
	N_{err}, bites	100000	100000	28751	147	0	0	0	0
	N_{tot}, bites	100000	100000	100000	100000	100000	100000	100000	100000

Table 6.6.

Modem study results

h^2	0,1	1,00	1,5	4,0	7,0	20,0	Note
σ_A	0,07	0,06	0,04	0,04	0,04	0,01	experiment
P_{cal}	0,5	0,3	$2{,}6\cdot10^{-3}$	$3{,}8\cdot10^{-11}$	$2{,}2\cdot10^{-13}$	$1\cdot10^{-45}$	Table 6.1
P_{ex}	0,5	0,36	$2{,}9\cdot10^{-3}$	no data	$5\cdot10^{-6}$	0	Table 6.1
P_{ex}	0,5	0,14	$1{,}4\cdot10^{-3}$	$7{,}5\cdot10^{-5}$	0	0	Table 6.5
P_{ex}	$4{,}7\cdot10^{-2}$	0	0	0	0	0	Table 6.2,6.3 sine channel
P_{ex}	0,5	$1\cdot10^{-5}$	0	0	0	0	Table 6.2,6.3 cosine channel
P_{ex}	$1{,}3\cdot10^{-2}$	0	0	0	0	0	Table 6.4
P_{cal}	0,5	0,1	$4{,}2\cdot10^{-3}$	$2{,}3\cdot10^{-3}$	$1\cdot10^{-4}$	$1\cdot10^{-10}$	QPSK

To compare the simulation results with the theoretical values of the error probability, graphs are plotted in Figure 6.9, which belong to signal modems with different types of modulation. Curve 1 is plotted for known 4-QAM modulation and curve 3 for known QPSK modulation. Curves 2, 4 – 7 are plotted for the new SSK modulation. Curve 2 shows the error probability of a single-channel modem with a non-optimal algorithm, and curve 4 with an optimal algorithm. Curve 5 refers to a two-channel modem (cosine channel); curve 7 - sinus channel. Curve 6 refers to a two-channel modem with channel bonding connected.

The limited amount of computer RAM made it possible to write only 10^7 discrete values of the signal into the workspace of the Matlab package and then transfer 10^5 binary elements. The marginal probability of errors in this case was 1·10-5. The calculated (total) probability of modem errors is much less than the level of $1\cdot10^{-15}$ and it is not possible to check it on the model. Indeed, in the cosine channel of the modem, we managed to check the probability of errors at the ratio $h^2 = 1$. It turned out to be equal to $1\cdot10^{-5}$ (Table 6.2) and coincided with the calculated value of the error probability (point D in Fig. 6.9) [24].

When modeling a modem with one channel, it was found that the mean root square (RMS) of the estimate of the real part of the Lyapunov ch.f. exceeds the previously known theoretical value σ_A at ratios $h^2<10$. In accordance with the work [2] σ_A=0,01. The new RMS values of the assessment are recorded in Table 6. 6.

Taking into account these data, the error probabilities of a single-channel modem were calculated for the set threshold $\Pi_{2к}$=0,6325 and the modulation algorithm recorded in Table 6.1. The order of the calculated and experimental error probabilities coincided with the ratio $0,1\leq h^2 \leq 1,5$.

At the end of the analysis, consider the optimal signal modulation algorithm (Table 4.11) in a modem with one channel, the data on which are recorded in Table 6.5. They are better than the data of the same name in Table 6.1 and confirm the results of the theory. For comparison, the last row of Table 6.6 shows the error probabilities of the QPSK signaling modem. Comparison of the digits of this line with the numbers of other lines of Table 6.6 generates a conclusion not in favor of QPSK modulation. It was theoretically found that the new SSK modulation in terms of energy performance, outgoes by 10 dB over QPSK modulation (compare curves 3 and 4).

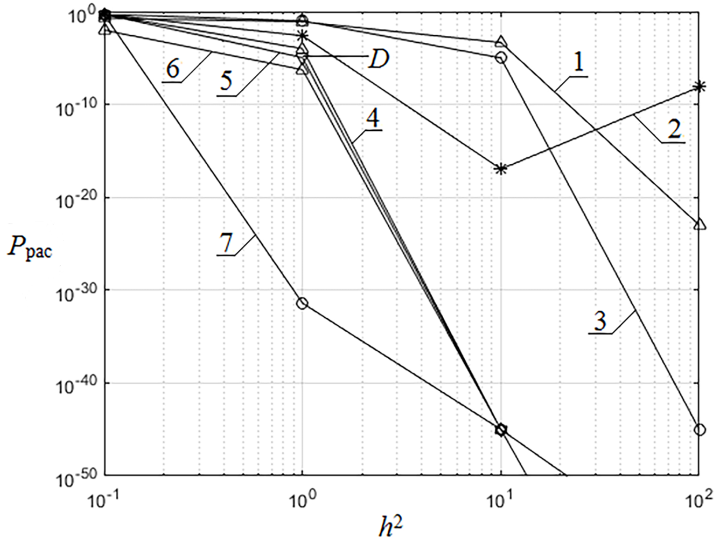


Figure 6.9. *Probability of errors of different modems*

Thus, the simulation results of the modem do not contradict the theoretical data regarding its performance, and they are better than those of the QPSK signal modem.

The simulation showed the feasibility of the practical implementation of the device and the operability of the new modem, and also provided a test of its characteristics when operating in a channel with "white" noise. The noise immunity of a modem of a single-channel, two-channel modem and a modem with a connected circuit for combining the outputs of the sine and cosine channels of the device is studied. At a signal-to-noise ratio of one or less, the noise immunity of the devices turned out to be different, and at large signal-to-noise ratios, there were no errors when receiving binary symbols. The minimum probability of modem errors is determined at the level of $1 \cdot 10^{-5}$ with the volume of received binary symbols 10^5. This volume of symbols is limited by the technical characteristics of the computer and could not be increased more. Nevertheless, in the error probability interval 1.0…$1 \cdot 10^{-5}$, the signal modem with SSK modulation has energy efficiency indicators better than all known devices.

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NOTES

NOTES

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THE FOUNDATIONS OF THE THEORY OF CONSTRUCTION OF NEW-GENERATION MODEMS

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