# Energetics of Pulse Cenerators with Current Interruption in an Inductive Store 

Loginov Sergei Vasilievich<br>Candidate of Physico-mathematical Sciences, Senior Research Officer Institute of High Current Electronics SB RAS<br>Tomsk, Russia


#### Abstract

The paper presents an electrotechnical analysis of circuits based on an inductive energy store and an opening switch for operation with no load, with an inductive load, and with a resistive load, and also with two-stage pulse sharpening and upstream-of-switch load connection. The function of a switch is to cut off the load of a pulse generator during energy storage and to provide fast energy delivery to the load on approaching a certain critical current. The analysis suggests simple and useful formulae to estimate the load pulse parameters at a linearly rising switch resistance. The approximation of a linear resistance rise at the phase of current cutoff is a useful tool to assess the energetics of pulse generators both with plasma opening switches and with exploding wire switches. The estimates are compared with experimental data. The use of a more complex resistance approximation can improve the agreement between calculations and experiments, but it inevitably deprives the relations of their simplicity, clarity, and promptitude.


## I. INTRODUCTION

Inductive energy stores, being ten times superior to capacitive ones in energy density, can greatly decrease the weight, dimensions, and cost of pulsed power systems [1]. However, their efficient use needs a switch capable of providing high-current interruption, many-fold pulse compression, and power amplification at a load. Among the most widespread types of switch are exploding wire switches [2], [3] and plasma opening switches [4], [5]. The function of a switch is to cut off the load of a pulse generator during energy storage and to provide fast energy delivery to the load on approaching a certain critical current.

This paper analyzes the operation of circuits based on an inductive energy store and an opening switch (shortly, inductive store-switch circuits) with no load, with an inductive load, and with a resistive load, and also with two-stage pulse sharpening and upstream-of-switch load connection. The analysis suggests simple and useful formulae to promptly estimate the load pulse parameters at a linearly rising switch resistance. The estimates and respective current and voltage waveforms are compared with experimental data.

## II. Inductive Store-Switch Circuits

Figure 1 shows three main circuits with an inductive energy store and a switch. When the switch is conducting, the inductance $L_{g}$ is charged from the primary capacitive energy store to a current $I_{0}$. The voltage at the instant of current cutoff depends on the circuit parameters and on the rate of rise of the switch resistance.

When the switch also serves as a load (Fig. 1a), the discharge current is determined by the equation $L_{g} \dot{I}_{s}(t)+R_{s}(t) I_{s}(t)=0$, where (like in all further expressions) the dot over the current
symbol stands for a time derivative. The solution of this equations has the form $I_{s}(t)=I_{0} \exp \left[-L_{g}^{-1} \int_{0}^{t} R_{s}\left(t^{\prime}\right) d t^{\prime}\right]$ with $I_{0}$ for the inductance current at the instant of switch operation. Because the current decays exponentially, the switch voltage $V_{s}(t)=I_{s}(t) R_{s}(t)$ reaches its peak subject to $\dot{R}_{s}(t)=R_{s}^{2}(t) / L_{g}$. For the resistance rising linearly $R_{s}(t)=\dot{R}_{s} t$ with a constant rate $\dot{R}_{s}=$ Const , the peak voltage

$$
\begin{equation*}
V_{m}=I_{0}\left(\dot{R}_{s} L_{g} / \mathrm{e}\right)^{1 / 2}, \tag{1}
\end{equation*}
$$

where $e$ is the base of natural logarithms, is attained at the point in time

$$
\begin{equation*}
t_{m}=\left(L_{g} / \dot{R}_{s}\right)^{1 / 2} \tag{2}
\end{equation*}
$$

The voltage full width at half maximum $\Delta t \approx 1.60 t_{m}$, where the numerical factor is equal to the difference of roots of the equation $2 x \mathrm{e}^{\left(1-x^{2}\right) / 2}=1, x=t / t_{m}$. Because the maximum energy store

current is $I_{0}=U_{0} / \rho$, where $U_{0}$ is the output voltage of the primary capacitive store with a discharge capacitance $C$ and $\rho=\left(L_{g} / C\right)^{1 / 2}$ is the wave impedance, the voltage multiplication factor according to (1) is $K=\left(\dot{R}_{s} C / \mathrm{e}\right)^{1 / 2}$.

The dissipated switch power reaches its extremum $P_{s \max }$ at the time point $t_{m}^{\prime}$ determined from the equation $\dot{R}_{s}(t)=2 R_{s}^{2}(t) / L_{g}$.


Fig. 1. Inductive store-switch circuits. From this we have $t_{m}^{\prime}=\left(L_{g} / 2 \dot{R}_{s}\right)^{1 / 2}$ and $P_{s \text { max }}=\left(L_{g} \dot{R}_{s} / 2 \mathrm{e}\right)^{1 / 2} I_{0}^{2}$. For $L_{g}=100 \div 400 n H, I_{0}=1 \mathrm{MA}$, and $\dot{R}_{s}=0.1 \Omega / \mathrm{ns}$, the voltage rises to $V_{m}=2 \div 4 \mathrm{MV}$ in $t_{m}=30 \div 60 \mathrm{~ns}$ at $P_{s \text { max }}=1.4 \div 2.8 \mathrm{TW}$.

When the switch operates into an inductance $L_{l}$ (Fig. 1b), the store current $I_{g}(t)$, the switch current $I_{s}(t)$, and the load current $I_{l}(t)$ are determined from the system of equations

$$
\left\{\begin{array}{l}
I_{g}(t)=I_{s}(t)+I_{l}(t) \\
L_{l} \dot{I}_{l}(t)=R_{s}(t) I_{s}(t) \\
L_{g} \dot{I}_{g}(t)+R_{s} I_{s}(t)=0
\end{array}\right.
$$

where the initial conditions are $I_{g}(0)=I_{s}(0)=I_{0}, I_{l}(0)=0$. Thus, we have the load current $I_{l}(t)=I_{0}\left[L_{g} /\left(L_{g}+L_{l}\right)\right]\left\{1-\exp \left[-L_{t}^{-1} \int_{0}^{t} R_{s}\left(t^{\prime}\right) d t^{\prime}\right]\right\} \quad$ and the switch current
$I_{s}(t)=I_{0} \exp \left[-L_{t}^{-1} \int_{0}^{t} R_{s}\left(t^{\prime}\right) d t^{\prime}\right]$, where $L_{t}=L_{g} L_{l} /\left(L_{g}+L_{l}\right)$. The parameters $V_{m}, t_{m}$ are given by (1) and (2) with $L_{g} \rightarrow L_{t}$. At $L_{g}=L_{l}$, these parameters decrease $\sqrt{2}$ times compared to their values for the circuit in Fig. 1a.

For full opening of the switch, the total energy in the circuit elements is $W_{f}=W_{0} L_{g} /\left(L_{g}+L_{l}\right)$, where $W_{0}=L_{g} I_{0}^{2} / 2$ is the stored energy. The ratio $W_{f} / W_{0}=L_{g} /\left(L_{g}+L_{l}\right)$ decreases from unity at $L_{l} \rightarrow 0$ to zero at $L_{l} \rightarrow \infty$. On the contrary, the dissipated energy $W_{R_{s}} / W_{0}=L_{l} /\left(L_{g}+L_{l}\right)$ increases from zero at $L_{l} \rightarrow 0$ to unity at $L_{l} \rightarrow \infty$. The load energy $W_{l}=W_{0} L_{g} L_{l} /\left(L_{g}+L_{l}\right)^{2}$ is maximal at $L_{g}=L_{l}$ for which $W_{l}=0.25 W_{0}$ and $W_{R_{s}}=0.5 W_{0}$.

The load pulse power $\quad P_{l}(t)=\left[L_{g} /\left(L_{g}+L_{l}\right)\right] R_{s}(t) I_{0}^{2} f(t)$, where $f(t)=\left\{1-\exp \left[-L_{t}^{-1} \int_{0}^{t} R_{s}\left(t^{\prime}\right) d t^{\prime}\right]\right\} \exp \left[-L_{t}^{-1} \int_{0}^{t} R_{s}\left(t^{\prime}\right) d t^{\prime}\right]$, is maximal at the time point $\tilde{t}_{m}$ determined from the equation $\mathrm{e}^{-\theta}=(1-2 \theta) /(1-4 \theta)$, where $\theta=\dot{R}_{s} \tilde{t}_{m}^{2} / 2 L_{t}$. From this we have $\theta \approx 1.06$, $P_{l \max }=\alpha\left(L_{t} \dot{R}_{s}\right)^{1 / 2} L_{g} I_{0}^{2} /\left(L_{g}+L_{l}\right)$, where $\alpha=(2 \theta)^{1 / 2} \mathrm{e}^{-\theta}\left(1-\mathrm{e}^{-\theta}\right)$. At $L_{l}=L_{g} / 2$, the power reaches its absolute extremum $\quad P_{l \max \left(L_{l}\right)}=\left(2 \alpha / 3^{3 / 2}\right)\left(L_{g} \dot{R}_{s}\right)^{1 / 2} I_{0}^{2}$. At $L_{g}=L_{l}$, the power is $P_{l \text { max }}=\left(\alpha / 2^{3 / 2}\right)\left(L_{g} \dot{R}_{s}\right)^{1 / 2} I_{0}^{2}$.

With a resistive load (Fig. 1c), the currents are determined from the equations

$$
\left\{\begin{array}{l}
I_{g}(t)=I_{s}(t)+I_{d}(t) \\
R_{d} I_{d}(t)=R_{s} I_{s}(t) \\
L_{g} I_{g}(t)+R_{s}(t) I_{s}(t)=0,
\end{array}\right.
$$

where the initial conditions are $I_{g}(0)=I_{s}(0)=I_{0}, I_{d}(0)=0$. For $R_{d}=$ Const , the switch current is

$$
\left.I_{s}(t)=I_{0} \exp \left[-\int_{0}^{t}\left\{\left[L_{g} \dot{R}_{s}\left(t^{\prime}\right)+R_{s}\left(t^{\prime}\right) R_{d}\right] / L_{g}\left[R_{s}\left(t^{\prime}\right)+R_{d}\right]\right\}\right] d t^{\prime}\right] .
$$

This current can be expressed as $I_{s}(t)=I_{0}\left\{R_{d} /\left[R_{s}(t)+R_{d}\right]\right\} \exp \left[-f(t) R_{d} / L_{g}\right]$, where $f(t)=\int_{0}^{t}\left\{\left[R_{s}\left(t^{\prime}\right) / R_{d}\right] /\left[1+R_{s}\left(t^{\prime}\right) / R_{d}\right]\right\} d t^{\prime} . \quad$ For $\quad R_{s}(t)=\dot{R}_{s} t, \quad$ we have the function $f(t)=t-k^{-1} \ln (1+k t)$ and the switch current is

$$
\begin{equation*}
I_{s}(t)=I_{0}(1+k t)^{(1-k \tau) / k \tau} \exp (-t / \tau) \tag{3}
\end{equation*}
$$

where $k=\dot{R}_{s} / R_{d}, \tau=L_{g} / R_{d}$.

The peak voltage across the load $V_{d}(t)$ is reached at the time point determined from (2), which is independent of $R_{d}$. However, the value of the peak $V_{d}(t)$ depends both on the load resistance $\propto R_{d}^{0.40 .5}$ (Fig. 2a, $L_{g}=200 \mathrm{nH}, I_{0}=1 \mathrm{MA}$ ) and on the switch resistance rise rate $\dot{R}_{s}$ (Fig. 2b). At $R_{d} \rightarrow \infty$, the voltage tends to $V_{m}=I_{0}\left(\dot{R}_{s} L_{g} / \mathrm{e}\right)^{1 / 2} \quad$ because, in view of (3), $V_{m}=I_{0}\left(\dot{R}_{s} L_{g}\right)^{1 / 2}\left(1+x^{-1}\right)^{x^{2}-1} \mathrm{e}^{-x}$, where $x=R_{d} /\left(\dot{R}_{s} L_{g}\right)^{1 / 2}$, and at $x \rightarrow \infty$, the limit is known and is $\lim _{x \rightarrow \infty}\left(1+x^{-1}\right)^{x^{2}-1} \mathrm{e}^{-x}=\mathrm{e}^{-1 / 2}$.



Fig. 2. Peak voltage $V_{d}\left(R_{d}\right)$ at $\dot{R}_{s}=0.2$ (1), 0.1 (2), 0.05 (3) $\Omega / \mathrm{ns}$ (a), and $V_{d}\left(\dot{R}_{s}\right)$ at $R_{d}=1(1), 2(2), 3(3) \Omega(\mathrm{b})$.


Fig. 3. Load and switch powers (a) and energies (b) versus $R_{d}$ at $\dot{R}_{s}=0.2$ (1), 0.1 (2), and 0.05 (3) $\Omega / \mathrm{ns}$.

As can be seen in Figure 2a, the voltage first rises quasi-linearly with the load resistance. Such a behavior can be interpreted as load-limited operation [6] in which the load current depends weakly on the load resistance. Further increasing the load resistance slows down the voltage rise, and this can be interpreted as switch-limited operation in which the load current is approximately inversely proportion to the load resistance. For the curves in Figure 2a, the conditional boundary between the two operation modes lies at $\sim 1.5 \Omega$.

The dissipated switch power reaches its peak at $t_{m}^{\prime \prime}=(\tau / 4)\left[(1+8 / k \tau)^{1 / 2}-1\right]$. The load power shows its absolute extremum $P_{d \text { max }}=\left(k t_{m}\right)^{2}\left(1+k t_{m}\right)^{2(1-k \tau) / k \tau}\left[\exp \left(-2 t_{m} / \tau\right)\right] R_{d} I_{0}^{2}$ at an optimum resistance $R_{\text {dopt }} \approx \beta\left(\dot{R}_{s} L_{g}\right)^{1 / 2}$. The coefficient $\beta=(k \tau)^{-1 / 2} \approx 0.62$ is the solution of the equation $4 \beta^{2} \ln \left(1+\beta^{-1}\right)=4 \beta-1$. For such a load, $I_{d \max } \approx 0.48 I_{0}, P_{d \text { max }} \approx 0.14\left(\dot{R}_{s} L_{g}\right)^{1 / 2} I_{0}^{2}$, and the peak voltage $V_{d \text { max }} \approx 0.30\left(\dot{R}_{s} L_{g}\right)^{1 / 2} I_{0}$ is two times lower than (1). The voltage FWHM is $\Delta t \approx 2.93 t_{m}$.

Here, the numerical factor is the difference of roots of the equation $[(a+1) /(a+x)]^{a^{2}-1} \mathrm{e}^{-a(1-x)}=2 x$, where $x=t / t_{m}, a=\left(k t_{m}\right)^{-1}$.

At other values of $R_{d}$, the energy characteristics behave as expected (Fig. 3). Increasing $R_{d}$ (e.g., at $L_{g}=200 \mathrm{nH}, I_{0}=1 \mathrm{MA}$ ) increases the dissipation in the switch and hence decreases the energy extraction. Increasing $\dot{R}_{s}$ increases the load energy. Because $R_{\text {dopt }} \approx \beta\left(\dot{R}_{s} L_{g}\right)^{1 / 2}$, its value tends to increase as $\dot{R}_{s}$ is increased (Fig. 3a).

For a load with $R_{d o p t}$, the overvoltage coefficient is $K=V_{d \max } / U_{0} \approx 0.30\left(\dot{R}_{s} C\right)^{1 / 2}$. For $K$ as a parameter, we have $R_{\text {dopt }} \approx 2.0 \mathrm{~K} \rho$, and

$$
\begin{equation*}
P_{d \text { max }} \approx 0.95 K W_{0} \omega_{0}, \tag{4}
\end{equation*}
$$

where $W_{0}=C U_{0}^{2} / 2, \rho=\left(L_{g} / C\right)^{1 / 2}, \omega_{0}=\left(L_{g} C\right)^{-1 / 2}$.
With no switch, the load current is $I(t)=\left[\left(U_{0} / \omega L_{g}\right) \exp \left(-R t / 2 L_{g}\right)\right] \sin \omega t$, where $\omega=\omega_{0}\left(1-v^{2}\right)^{1 / 2}$, and the damping decrement $v=R / 2 \rho$ is less than critical [7]. The current amplitude $I_{m}=I_{0} \exp \left\{-v\left(1-v^{2}\right)^{-1 / 2} \operatorname{arctg}\left[\left(1-v^{2}\right)^{1 / 2} / v\right]\right\}$, where $I_{0}=U_{0} / \rho$, is attained at the time point $t_{m}=\omega_{0}^{-1}\left(1-v^{2}\right)^{-1 / 2} \operatorname{arctg}\left[\left(1-v^{2}\right)^{1 / 2} / v\right]$. The load power is maximal at $v_{\text {opt }}=0.55$, which is the solution of the equation $2 v \operatorname{arctg}\left[\left(1-v^{2}\right)^{1 / 2} / v\right]=\left(1-v^{2}\right)^{1 / 2}\left(1+v^{2}\right)$. The peak power

$$
\begin{equation*}
P_{\mathrm{m}} \approx 0.60 W_{0} \omega_{0} \tag{5}
\end{equation*}
$$

is dissipated at $R_{o p t} \approx 1.1 \rho$. From comparison of (4) and (5) it follows that the use of a switch which provides an overvoltage $K \sim 10$ can increase the load power about 15 times.

## III. Two-Stage Pulse Sharpening

Figure 4 shows two series-connected inductive store-switch circuits. On opening of the first switch $R_{s 1}$ with the second switch $R_{s 2}$ closed, the energy from $L_{g}$ is extracted to $L_{\text {int }}$. On opening of $R_{s 2}$ the energy is switched from $L_{\text {int }}$ to the load. Increasing the rate of rise of the current in the switch $R_{s 2}$ increases the rate of rise of its resistance and hence the output generator voltage and the load power. For example, in experiments [8], the voltage reached $\sim 1 \mathrm{MV}$ on operation of the first plasma opening switch (conduction time $t_{c} \approx 1.2 \mu \mathrm{~s}$, conduction current $I_{c} \approx 1.7 \mathrm{MA}$ ), and on operation of the second one ( $t_{c} \approx 130 \mathrm{~ns}, I_{c} \approx 0.55 \mathrm{MA}$ ), the voltage across the diode load reached $\sim 4 \mathrm{MV}$. The resistance $R_{s 1}$ rose to about $1 \div 1.5 \Omega$ in 50 ns , and $R_{s 2}$ to about $10 \div 15 \Omega$ in 10 ns .

For an inductive load with fully open switches, the intermediate store current is $I_{\text {int }}=L_{g} I_{0} /\left(L_{g}+L_{\text {int }}\right)$ and the load current is $I_{l}=L_{\text {int }} L_{g} I_{0} /\left[\left(L_{\text {int }}+L_{l}\right)\left(L_{g}+L_{\text {int }}\right)\right]$. The maximum load current $I_{l \max }=I_{0}\left[1+\left(L_{l} / L_{g}\right)^{1 / 2}\right]^{-2}$ is reached at $L_{\mathrm{int}}=\left(L_{g} L_{l}\right)^{0.5}$. The load energy $W_{l}=W_{0} L_{g} L_{\mathrm{int}}^{2} L_{l}\left[\left(L_{g}+L_{\mathrm{int}}\right)\left(L_{\mathrm{int}}+L_{l}\right)\right]^{-2}$ with $W_{0}=L_{g} I_{0}^{2} / 2$ is maximal at $L_{g}=L_{\mathrm{int}}=L_{l}$, measuring $W_{l} / W_{0}=1 / 16$ or $6.25 \%$ of the inductive store energy. This casts doubts on the efficiency of multistage pulse sharpening for liner loads.

The switch voltage ratio for linearly rising switch resistances $\left(V_{s 2} / V_{s 1}\right)_{\max }=\left\{\left(\dot{R}_{s 2} / \dot{R}_{s 1}\right)\left[L_{g} L_{l} /\left(L_{g}+L_{\text {int }}\right)\left(L_{i n t}+L_{l}\right)\right]\right\}^{1 / 2}$ at $L_{g}=L_{\text {int }}=L_{l}$ is equal to $\left(\dot{R}_{s 2} / \dot{R}_{s 1}\right)^{1 / 2} / 2$. Hence, for increasing the load voltage, the rate of rise of the resistance $\dot{R}_{s 2}$ should be four times higher than that of $\dot{R}_{s 1}$.


Fig. 4. Two-stage pulse sharpening circuit.


Fig. 5. Dependences $\eta(L)$ at $L_{l}=20$

$$
\begin{aligned}
& \text { (1), } 200 \quad(2) \text {, and } 400 \text { (3) } \mathrm{nH} . \\
& L_{g}=200 \mathrm{nH} .
\end{aligned}
$$

The peak load power $P_{l \max }=\alpha\left[L_{\mathrm{int}}^{3} L_{l} /\left(L_{\mathrm{int}}+L_{l}\right)^{3}\right]^{1 / 2}\left[L_{g} /\left(L_{g}+L_{\mathrm{int}}\right)\right]^{2} \dot{R}_{s 2}^{1 / 2} I_{0}^{2}$ is attained at the time point $t_{m}=\left(2 \theta L_{t} / \dot{R}_{s}\right)^{1 / 2}$ with $L_{t}=L_{\text {int }} L_{l} /\left(L_{\mathrm{int}}+L_{l}\right)$ and with $\theta$ and $\alpha$ being the same as those for the one-stage circuit in Fig. 1b. The absolute extremum $P_{l \max \left(L_{\text {mit }}, L_{l}\right)}=(\alpha / 8)\left(\dot{R}_{s} L_{g}\right)^{1 / 2} I_{0}^{2}$ falls on $L_{\mathrm{int}}=L_{g} / 3, L_{l}=L_{g} / 6$ but the energy extraction efficiency, in this case, decreases to $1 / 24$. At $L_{g}=L_{\mathrm{int}}=L_{l}$, the peak power is equal to $P_{l \max }=(\alpha / 8 \sqrt{2})\left(\dot{R}_{s} L_{g}\right)^{1 / 2} I_{0}^{2}$. The power ratio for the twoand one-stage circuits is $\eta=0.25\left(\dot{R}_{s 2} / \dot{R}_{s 1}\right)^{1 / 2}$. Thus, for the most efficient energy extraction, the second stage is reasonable only if $\dot{R}_{s 2} / \dot{R}_{s 1}=16$.

For other values of $L_{g}, L_{\text {int }}, L_{l}$, the power ratio is given by $\eta=\left(\dot{R}_{s 2} / \dot{R}_{s 1}\right)^{1 / 2} \eta(L)$, where $\eta(L)=L_{\mathrm{int}}\left(L_{g} L_{l}\right)^{1 / 2}\left(L_{g}+L_{\mathrm{int}}\right)^{-1 / 2}\left(L_{\mathrm{int}}+L_{l}\right)^{-3 / 2}$, and its extremum is reached at $L_{\mathrm{int}}=\left[\left(L_{g}-L_{l}\right)^{2} / 16+L_{g} L_{l}\right]^{1 / 2}-\left(L_{g}-L_{l}\right) / 4$ (Fig. 5).

## IV. UpsREAM-OF-SWITCH LOAd CONNECTION

One of the ways of eliminating the adverse effect of plasma from switch to load is to connect the load upstream of the switch [9]. Such a circuit (Fig. 6) includes a separating closing switch $S w$
which cuts off the load from current till the opening switch gets open. Let the switch $S w$ turns on at the onset of current interruption, and after fast switching, its resistance drops to zero. Before opening, the currents through the inductances $L_{s}$ and $L_{g}$ are equal to $I_{0}$. After opening, the currents are determined from the system of equations

$$
\left\{\begin{array}{l}
L_{g} \dot{I}_{g}(t)+L_{s}\left[\dot{I}_{g}(t)-\dot{I}_{l}(t)\right]+R_{s}(t)\left[I_{g}(t)-I_{l}(t)\right]=0 \\
L_{g} \dot{I}_{g}(t)+L_{l} \dot{I}_{l}(t)=0
\end{array}\right.
$$

Solving the system gives the same time dependences as those for the circuit in Fig. 1b but with $L_{t}=L_{s}+L_{g} L_{l} /\left(L_{g}+L_{l}\right)$. The presence of $L_{s}$ increases $t_{m}=\left(L_{t} / \dot{R}_{s}\right)^{1 / 2}$ and decreases the peak voltage across the load $V_{l \max }=I_{0}\left(\dot{R}_{s} / \mathrm{e} L_{t}\right)^{1 / 2} L_{g} L_{l} /\left(L_{g}+L_{l}\right)$. At $L_{l}=L_{g}$, the peak voltage $V_{l \max }$ is $\left(1+2 L_{s} / L_{g}\right)^{-1}$ of the switch voltage. The maximum load power is $P_{l \max }=\lambda \dot{R}_{s}^{1 / 2} I_{0}^{2}$, where $\lambda=\alpha L_{g}^{2} L_{l}\left(L_{g}+L_{l}\right)^{-3 / 2}\left[\left(L_{g}+L_{l}\right) L_{s}+L_{g} L_{l}\right]^{-1 / 2}$.


Fig. 6. Upstream-of-switch load connection.


Fig. 7. Dependences $\lambda\left(L_{l}\right)$ at $L_{s}=10$ (1), 50 (2), and 100 (3) nH. $L_{g}=200 \mathrm{nH}$.

The dependences $\lambda\left(L_{l}\right)$ at different values of $L_{s}$ are shown in Figure 7. As can be seen, varying $L_{s}$ by an order of magnitude influences $\lambda$ only slightly. At the same time, the presence of $L_{s}$ greatly decreases the load power at a load inductance of less than 100 nH . The power $P_{l \max }$ reaches its absolute extremum at $L_{l}=\varphi\left(L_{s} / L_{g}\right) L_{g}^{2} / 4\left(L_{g}+L_{s}\right)$, where $\varphi(x)=1+[1+16 x(1+x)]^{1 / 2}$. At $L_{s} \square L_{g}$, we have $L_{l}=L_{g} / 2$ and $P_{l \max \left(L_{l}\right)}=\left(2 \alpha / 3^{3 / 2}\right)\left(L_{g} \dot{R}_{s}\right)^{1 / 2} I_{0}^{2}$.

The ratio of the load energy $W_{l}=W_{0} L_{g}^{2} L_{l} /\left[\left(L_{g}+L_{l}\right)^{2}\left(L_{g}+L_{s}\right)\right]$, where $W_{0}=\left(L_{g}+L_{s}\right) I_{0}{ }^{2} / 2$, to its value in the conventional circuit is $\left(1+L_{s} / L_{g}\right)^{-1}$. At $L_{g} / L_{s} \sim 10$, the decrease in the energy extraction efficiency is less than $\sim 10 \%$.

## V. COMPARISON WITH EXPERIMENTS

According to empirical data [10], the voltage multiplication factor provided by an exploding wire switch at a load with $R \approx 28 \rho$ is $K \approx 13$. For this value of $K$, the optimum resistance is
$R_{\text {dopt }} \approx 26 \rho$. The fact that $R_{\text {dopt }}$ is close to $R$ justifies the assumption of a linear resistance rise on current cutoff for calculations of the load pulse parameters.

Let us refer to experimental data. Figure 8 shows waveforms recorded on operation of one of the generators built around an inductive energy store and a exploding wire switch for high-power microwave sources [11]. The store, having an inductance of $\sim 4.6 \mu \mathrm{H}$, and the switch represent a series connected circuit into which a capacitor of $\sim 4 \mu \mathrm{~F}\left(U_{c h}=70 \mathrm{kV}\right)$ is switched. The voltage $U_{s}$ that arises on interruption of the current $I_{g} \approx 30 \mathrm{kA}$ is applied via a sharpening switch to a resistive load of $\sim 40 \Omega$. The rate of rise of the load current is $\sim 0.5 \mathrm{kA} / \mathrm{ns}$, the voltage is $U_{l} \approx 600 \mathrm{kV}$, and the pulse power is $P_{l} \approx 9 \mathrm{GW}$.


Fig. 8. Shot with wire switch. Interpretation of traces is given in respective paragraphs.


Fig. 9. Load voltage (a) and switch and load currents (b).

Figure 9 compares the experimental and calculated waveforms of the load voltage, switch current, and load current. The calculation is by formula (3) for $\dot{R}_{s} \approx 1.7 \Omega / \mathrm{ns}$, which approximates the steep rise of resistance on current cutoff. As can be seen, the experimental and calculated current waveforms, being coincident in amplitude, show some difference during the pulse rise time, and this is due to the transient response of the sharpening switch whose resistance decreases to $\sim 1.5 \Omega$ only within $\sim 60 \mathrm{~ns}$ after the onset of current flow through the load. The finite resistance of this switch delays the extraction of energy. The increase in $I_{\text {sexp }}(t)$ within $\sim 100 \mathrm{~ns}$ after the instant of current cutoff suggests that the exploding wire switch recovers its conductivity, which shows up as a decay of $R_{s}(t)$ in Figure 8.

The assumption of a linear resistance rise can also be justified by the example of experiments on GIT-4 and GIT-12 mega-joule setups with mega-ampere plasma opening switches [10]. In the GIT4 setup, the stored energy is $\sim 550 \mathrm{~kJ}$ at a charge voltage $U_{c h}=40 \mathrm{kV}$; the discharge current rises to $\sim 3 \mathrm{MA}$ in $\sim 1.1 \mu \mathrm{~s}$. In the GIT- 12 setup, the stored energy is $\sim 5 \mathrm{MJ}$ at $U_{c h}=70 \mathrm{kV}$; the current rises to $\sim 6 \mathrm{MA}$ in $\sim 1.7 \mu \mathrm{~s}$.

Figure 10a shows waveforms for one of the shots on the GIT-12 setup with a radial plasma opening switch [12]. In its energy store with $\sim 160 \mathrm{nH}$, the current $I_{g}$ reaches $\sim 1.2$ MA in $\sim 550 \mathrm{~ns}$.

The switch operates into a load of $\sim 50 \mathrm{nH}$. On current cutoff, the switch resistance increases with a rate of $\sim 17 \mathrm{~m} \Omega / \mathrm{ns}$. As can be seen in Figure 10 b , the curves $U_{\text {cal }}(t)$ and $U_{\text {cal2 }}(t)$ calculated for this value of $\dot{R}_{s}$ and for its two times lower value lie above and below the amplitude of the experimental curve $U_{s}(t)$.


Fig. 10. Shot on GIT-12 setup at $L_{l} \approx 50 \mathrm{nH}$ with $I_{s}$ for switch current.


Fig. 11. Shot on GIT-4 setup.

Figure 11a shows waveforms for one of the shots on the GIT-4 setup in open-cathode mode [12]. In its energy store with $\sim 270 \mathrm{nH}$, the current reaches $I_{s} \approx 1 \mathrm{MA} \mathrm{in} \sim 800 \mathrm{~ns}$. The voltage that arises on current cutoff is $U_{s} \approx 2 \mathrm{MV}$. The switch resistance $R_{s}$ increases with a rate of $\sim 85 \mathrm{~m} \Omega / \mathrm{ns}$ (Fig. 11b). At this value of $\dot{R}_{s}$, the switch fully open into a load with an unlimited impedance would provide a peak voltage of $U_{\text {call }}(t) \sim 3 \mathrm{MV}$. However, in the shot considered, the increment of the discharge circuit inductance after opening of the switch is limited to $L_{x} \approx 230 \mathrm{nH}$. For this load, the calculated voltage $U_{\text {cal } 2}(t)$ has its amplitude close to the peak of $U_{s}(t)$. The current $I_{\text {scal }}(t)$ calculated for experimental values of $R_{s}(t)$ and finite value of $L_{x}$ decays more appreciably than the switch current $I_{s}(t)$, suggesting that $I_{s}(t)$ is prevented from decay by certain processes which are discussed, e.g., elsewhere [13-15].

The foregoing demonstrates that the approximation of a linear resistance rise at the phase of current cutoff is a useful tool to assess the energetics of pulse generators both with plasma opening switches and with exploding wire switches. Certainly, the use of a more complex resistance approximation can improve the agreement between calculations and experiments, but it inevitably deprives the relations of their simplicity, clarity, and promptitude.

## VI. Conclusion

Thus, the electrotechnical analysis of inductive store-switch circuits provides simple analytical formulae which are useful both for experimental data interpretation and for predication of load pulse parameters in generators based on this type of circuit. The load pulse parameters depend on
the rate of rise of the switch resistance. For its estimation, which is left untouched in the paper, one should consider physical processes responsible for current interruption in one or another switch. For example, as applied to plasma opening switches, such a consideration gives $\dot{R}_{s}$ as a function of the switch parameters and rate of current rise [16]. As applied to exploding wire switches, the rate of rise of the resistance can be estimated from empirical similarity criteria [10].

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