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Mathematical models of propagation and reflection of a plastic wave in a band lying on an elastic half-space, which has the property of linear compressibility and linear irreversible unloading [1]

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Abstract. The problem of the effect of a mobile load on a soil layer of finite thickness lying on a horizontal elastic foundation is considered.

The soil is modeled by an ideal nonlinearly compressible and irreversible unloading medium, in which the relationship between pressure and volumetric deformation under loading and during unloading of the medium is linear and irreversible.

The load is applied to the upper surface of the layer and moves at a superseismic speed. The problem of the effect of a moving load on a two-layer medium consisting of a soft soil layer and an elastic-yielding pad with different thicknesses and densities is considered. The solution to the problem is constructed analytically in both reverse and direct ways.

A two-layer medium consists of a soft soil layer of thickness *h* with an elastic deformable base. The soil is modeled by an inelastic ideal medium with linear compressibility and linear irreversible unloading. Consequently, the shear resistance of the medium is neglected. According

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to this statement, the influence of the deformability of the base and the load profile on the distribution of the dynamic parameters of the layer and the contact surface was investigated.

Keywords. Mathematical models, propagation of a plastic wave, half-space, analytical solution, wave front, ideal fluid, linear compressibility, irreversible unloading. equation of motion, continuity, states of the environment

Formulation of the problem. Let us consider the problem of the propagation of a plastic wave in a two-layer medium with a plane-parallel interface under the action of an intense load of a falling profile moving along its upper boundary with a constant super seismic velocity *D*.

A two-layer medium consists of a soft soil layer of thickness *h* with an elastic deformable base. The soil is modeled by an inelastic ideal medium with linear compressibility and linear irreversible unloading. Consequently, the shear resistance of the medium is neglected.

According to this statement, the influence of the deformability of the base and the load profile on the distribution of the dynamic parameters of the layer and the contact surface was investigated. The results of the numerical calculation are compared with the results of the acoustic layer and the layer with a rigid base. The solution of the problem is constructed in series, and their convergence is proved.

Let a monotonically decreasing normal load move along the upper boundary of the layer with an elastic base with a speed *D* exceeding the speed of wave propagation. The layer material has such a property that, under loading and unloading, the relationship between pressure P and volumetric deformation ε is linear and irreversible, the slope E_2 of the unloading branch of the $P - \varepsilon$ diagram exceeds the slope E_1 of the loading branch, i.e. $E_1 < E_2$.

Under the action of the above load, a compression wave Σ_1, first propagates in the layer, which is reached by the contact line of the media, induces a reflected plastic wave Σ_2, in the layer, and in the second medium a system of elastic (longitudinal and transverse) waves Σ_a and Σ_b. At $E_1 < E_2$ he speed of propagation of the AD characteristic is greater than the speed of the front Σ_2, therefore, as in the previous section, regions 2, 3, 4, etc. appear. On the Σ_a and Σ_b system, the layer material is instantly loaded, and then in areas 1, 2, 3, the medium is unloaded. Taking into account that the solution of the problem in domains 1 and 2 was obtained in the previous section, below we propose a solution to the problem only in domains 3 of layers and a, b of the elastic half-plane. For the joint problem of the domain 3, a, b (5.1.7) holds and the equations for the displacement potentials Φ, $\bar{\Psi}$ the elastic half-plane

$$\mu_1^2\frac{\partial^2\Phi}{\partial\xi^2}=\frac{\partial^2\Phi}{\partial\eta^2}\qquad,\mu_2^2\frac{\partial^2\Psi}{\partial\xi^2}=\frac{\partial^2\Psi}{\partial\eta^2}, \qquad (1)$$

$$\mu_1^2=\left(\frac{D}{a_0}\right)-1,\quad \mu_2^2=\left(\frac{D}{b_0}\right)-1,\quad a_0^2=\frac{\lambda+2G}{\rho_0},\quad b_0^2=\frac{G}{\rho_0}.$$

and according to the d'Alembert formula, their solutions are represented in the form

$$\begin{aligned}&\varphi(\xi,\eta)=f_3(\xi-\mu\eta)+f_4(\xi+\mu\eta),\quad \Phi(\xi,\eta)=F_3(\xi-\mu_1\eta),\\&\Psi(\xi,\eta)=F_2(\xi-\mu_2\eta)\end{aligned}. \qquad (2)$$

where $\rho_{02},\lambda,G-$ initial density and Lame coefficients of an elastic medium.

The boundary conditions for this problem are as follows:

at the front of the reflected wave at $\eta+\xi tg\alpha=2h$

$$tg\alpha\left(\vartheta_3^*-\vartheta_2^*\right)=u_3^*-u_2^*, \qquad (3)$$

on contact *AE* of two media at $\eta=h,\quad \xi\geq\frac{h}{tg\alpha}$

$$\sigma_{\xi\eta}=0,\quad D\left(\frac{\partial^2\Phi}{\partial\xi\partial\eta}+\frac{\partial^2\Psi}{\partial\xi^2}\right)=\frac{\partial\varphi}{\partial\xi},\quad P=-\sigma_{\eta\eta}. \qquad (4)$$

Here, $\sigma_{\xi\eta},\sigma_{\eta\eta}-$ stress components in an elastic medium. To find the function $f_4'(t)$ from (3) and (4), taking into account (2), we obtain the functional equation

$$f_4'(\xi)-\lambda_1 f_4'(\lambda_0\xi+2\mu h)=-\frac{\lambda_1}{\lambda_0}G_1(\xi), \qquad (5)$$

where

$$G_1(\xi)=f_1'(\xi-2\mu h)+\lambda_0 f_2'(\lambda_0\xi+2\mu h),$$

$$A(\lambda,G)=-\left[\lambda\frac{(\mu_1^2+1)(\mu_2^2-1)}{2\mu_1}+2G\left(\frac{\mu_1(\mu_2^2-1)}{2}+\mu_2\right)\right],$$

$$\frac{\lambda_1}{\lambda_2}=-\frac{A(\lambda,G)+\frac{\rho_0D^2}{\mu}\left(1+\frac{\mu_2^2-1}{2}\right)}{A(\lambda,G)-\frac{\rho_0D^2}{\mu}\left(1+\frac{\mu_2^2-1}{2}\right)}.$$

The solution to equation (5) is constructed by the method of successive approximations. Indeed, taking as the zero approximation

$$f_{40}'(\xi)=-\frac{\lambda_1}{\lambda_0}G_1(\xi).$$

for the first approximation we have

$$f_{40}'(\xi) = -\frac{\lambda_1}{\lambda_0}\left[G_1(\xi) + \lambda_1 G_1(\lambda_0\xi + 2\mu h)\right].$$

Then, continuing the iteration process, we obtain a recurrent formula of the form

$$f_4'(\xi) = -\frac{\lambda_1}{\lambda_0}\left[G_1(\xi) + \sum_{n=1}^{\infty}\lambda_1^n G\left(\lambda_0^n\xi + 2\mu h\frac{(\lambda_0^n - 1)}{(\lambda_0 - 1)}\right)\right]. \qquad (6)$$

Research has shown that $\lambda_1 << 1, \lambda_0 < 1$ and $G_1(\xi)$ are monotonically decreasing functions.

Consequently, according to the d'Alembert criterion, series (6) converges absolutely, and one can set the radius of its convergence. Then the solution of the problem taking into account (6) takes the form

$$P(\xi,\eta) = -\rho_0 D\Psi_{28}(\xi,\eta), \qquad (7)$$

$$\vartheta(\xi,\eta) = -\mu\Psi_{29}(\xi,\eta). \qquad (8)$$

where

$$\Psi_{28}(\xi,\eta) = \left\{\begin{array}{l} G_1(\xi - \mu\eta + 2\mu h) - \frac{\lambda_1}{\lambda_0}G_1(\xi + \mu\eta) + \\ +\sum_{n=1}^{\infty}\lambda_1^n G_1\left[\lambda_0^n(\xi - \mu\eta + 2\mu h) + 2\mu h\frac{(\lambda_0^n - 1)}{(\lambda_0 - 1)}\right] - \\ -\frac{\lambda_1}{\lambda_0}\sum_{n=1}^{\infty}\lambda_1^n G_1\left[\lambda_0^n(\xi + \mu\eta) + 2\mu h\frac{(\lambda_0^n - 1)}{(\lambda_0 - 1)}\right] \end{array}\right\},$$

$$\Psi_{29}(\xi,\eta) = \left\{\begin{array}{l} G_1(\xi - \mu\eta + 2\mu h) - \frac{\lambda_1}{\lambda_0}G_1(\xi + \mu\eta) + \\ +\sum_{n=1}^{\infty}\lambda_1^n G_1\left[\lambda_0^n(\xi - \mu\eta + 2\mu h) + 2\mu h\frac{(\lambda_9^n - 1)}{(\lambda_0 - 1)}\right] + \\ +\frac{\lambda_1}{\lambda_0}\sum_{n=1}^{\infty}\lambda_1^n G_1\left[\lambda_0^n(\xi + \mu\eta) + 2\mu h\frac{(\lambda_0^n - 1)}{(\lambda_0 - 1)}\right] \end{array}\right\}$$

In this case, the normal stress $\sigma_{\eta\eta}$ of the elastic half-plane in the regions a and b is determined by the formulas

$$\sigma_{\eta\eta}=\frac{\mu\left(\mu_2^2-1\right)\left(1+\dfrac{\lambda_1}{\lambda_0}\right)}{2D\mu_1\left(1+\dfrac{\mu_2^2-1}{2}\right)}\Psi_{30}(\xi,\eta)+\frac{2G\mu_2\mu\left(1+\dfrac{\lambda_1}{\lambda_0}\right)}{D\left(1+\dfrac{\mu_2^2-1}{2}\right)}\Psi_{31}(\xi,\eta), \quad (5.5.9)$$

at $\xi-\mu_2\eta\geq 0, \quad \eta\geq h.$

$$\sigma_{\eta\eta}=\frac{\mu\left(\mu_2^2-1\right)\left(1+\dfrac{\lambda_1}{\lambda_0}\right)}{2D\mu_1\left(1+\dfrac{\mu_2^2-1}{2}\right)}\Psi_{32}(\xi,\eta), \quad (10)$$

At $\xi-\mu_2\eta\geq 0, \quad \mu=h$,

$$\Psi_{30}(\xi,\eta)=\left\{\begin{array}{l}\left[\lambda\left(\mu_2^2+1\right)+2G\mu_1^2\right]G_1\left[\left(\xi-\mu_1^2\eta\right)+(\mu_1+\mu)h\right]+\\ +\sum_{n=1}^{\infty}\lambda_0^nG_1\left[\lambda_0^n\left(\xi-\mu_1\eta+(\mu_1+\mu)h\right)+2\mu h\dfrac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]\end{array}\right\},$$

$$\Psi_{31}(\xi,\eta)=\left\{\begin{array}{l}G_1\left[\left(\xi-\mu_2^2\eta\right)+(\mu_2+\mu)h\right]+\\ +\sum_{n=1}^{\infty}\lambda_1^nG_1\left[\lambda_0^n\left((\xi-\mu_2\eta)+(\mu_2+\mu)h\right)+2\mu h\dfrac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]\end{array}\right\},$$

$$\Psi_{32}(\xi,\eta)=\left\{\begin{array}{l}\left[\lambda\left(\mu_1^2+1\right)+2G\mu_1^2\right]G_1\left[(\xi-\mu_1\eta)+(\mu_1+\mu)h\right]+\\ +\sum_{n=1}^{\infty}\lambda_1^nG_1\left[\lambda_0^n\left(\xi-\mu_1\eta+(\mu_1+\mu)h\right)+2\mu h\dfrac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]\end{array}\right\},$$

If $\lambda\to\infty, \quad G\to\infty,$ then $\lambda_1=-\lambda_0$, and from (7), (8) for the case of a layer with an absolutely rigid base we have $P(\xi,\eta)=-\rho_0D\Psi_{33}(\xi,\eta)$, (11)

$$\vartheta(\xi,\eta)=-\mu\Psi_{34}(\xi,\eta). \quad (12)$$

where

$$\Psi_{34}(\xi,\eta)=\left\{\begin{array}{l}G_1\left[(\xi-\mu\eta+2\mu h)-G_1(\xi+\mu\eta)\right]+\\ +\sum_{n=1}^{\infty}(-\lambda_0)^nG_1\left[\lambda_0^n(\xi-\mu\eta+2\mu h)+2\mu h\dfrac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]-\\ -\sum_{n=1}^{\infty}(-\lambda_0)^nG_1\left[\lambda_0^n(\xi+\mu\eta)+2\mu h\dfrac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]\end{array}\right\},$$

$$\Psi_{33}(\xi,\eta)=\left\{\begin{array}{l} G_1(\xi-\mu\eta+2\mu h)-G_1(\xi-\mu_1\eta)+ \\ +\sum_{n=1}^{\infty}(-\lambda_0)^n G_1\left[\lambda_0^n(\xi-\mu\eta+2\mu h)+2\mu h\frac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]+ \\ +\sum_{n=1}^{\infty}(-\lambda_0)^n G_1\left[\lambda_0^n(\xi+\mu\eta)+2\mu h\frac{(\lambda_0^n-1)}{(\lambda_0-1)}\right]+ \end{array}\right\},$$

In the future, based on formula (7)-(12), it is necessary to carry out some calculations on a PC and analyze them.

Note that the above technique allows us to solve the problem of the effect of a moving load on a nonlinearly compressible strip lying on an elastic half-space.

Conclusion. The problem of propagation, reflection and a two-dimensional stationary plastic wave in a two-layer medium with densities ρ_1, ρ_2 is investigated for the case when the state diagram $P=P(\varepsilon)$ of the first medium (soil) is shock and under loading has the form $P(\varepsilon)=\alpha_1\varepsilon+\alpha_2\varepsilon^2$, and the second medium (black rock of a rock or pad) - elastic or rigid plastic. The problem is solved analytically by both direct and inverse methods, taking into account wave processes in the second medium and without taking them into account. Analysis of the results obtained on the PC shows that at $\rho_1>\rho_2$ taking into account the elastic - plastic properties of the second medium (spacer), modeled by a half-space, leads mainly to a decrease in the maximum values of stresses (pressure) at the contact of two media. At $\rho_1<\rho_2$ a stress concentration appears on the contact surface, and the pressure acquires the highest value in the case of an acoustic layer lying on a rigid foundation. The qualitative and quantitative picture of changes in the values of pressure and kinematic parameters depends not only on the stiffness characteristics of the media, but also on the ratio of their densities.

Thus, the above studies on the study of the two-dimensional stress-strain state of a homogeneous, inhomogeneous and layered medium under intense action of a mobile load on the boundary of a multilayer half-space confirm the need and importance of taking into account nonlinear, irreversible, wave processes.

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