УДК 531:536.66:
МРНТИ 27.35.31: 30.19.25,27,29,51,55,57.

Mathematical models of the propagation of a plastic wave in a half-space with the property of linear compressibility and linear irreversible unloading[1]

Aydosov Allayarbek
Academician of RANS, Doctor of Technical Sciences, Full Professor
Head of grant financing of the project APO9562377 RGP in PVKH "Institute of Information and Computing Technologies" SC MES RK. Almaty, Kazakhstan.
Aidosov Galym Allayarbekovich
Academician of RANS, Doctor of Technical Sciences, Full Professor
First Deputy General Director of KazMunayGas Aimak
Nur-Sultan, Kazakhstan.

Abstract. The problem of the effect of a moving load of an arbitrarily decreasing profile on a soil layer of finite thickness lying on a horizontal foundation is considered.

The soil is modeled by an ideal compressible medium in which the relationship between pressure and volumetric deformation under loading and during unloading of the medium is linear and irreversible.

The load is applied to the upper surface of the layer and moves at a superseismic speed.

If the moving load acting on the boundary of the half-space has a monotonically decreasing profile, then in the perturbation region, the medium is unloaded and the oblique compression wave is obtained by the load-unloading wave. The pressure of the medium against the background of this wave, depending on the depth of the half-space, decreases slowly than on the free surface. In the case when the relationship between pressure and deformation during loading of the medium is assumed to be nonlinear and shock, which corresponds to the propagation of a shock wave in the medium, the pressure in the perturbed region is somewhat overestimated in comparison with the linear one.

Keywords. Mathematical models, moving load, propagation, plastic wave, soil, half-space, wave front, ideal fluid, linear compressibility, irreversible unloading. equation of motion, continuity, states of the environment,

[1] **The work was carried out under the program APO9562377 of the Grant financing of SC MES RK**

Formulation of the problem. The problem of the effect of a moving load of an arbitrarily decreasing profile on a soil layer of finite thickness h, lying on a rigid horizontal foundation is considered.

The soil is modeled by an ideal compressible medium, in which the relationship between pressure P and volumetric deformation ε under loading and during unloading of the medium is linear and irreversible.

The load is applied to the upper surface of the layer and moves at a superseismic speed D. Since in this case the modulus of volumetric compression $\alpha_1 > E_1 -$of the modulus of unloading of the medium, in the physical plane (ξ,η) the characteristic AB has a greater in comparison with the speed of the reflected wave AD, and as a result, regions 2, 3, 4 appear, which are separated by the characteristic of the positive direction BC and the front reflected wave AD. The parameters of the environment in region 1 are known from the solution of the problem about AB. Note that this problem is stationary, and therefore all the parameters of the medium depend on two moving coordinates $\xi = x + Dt, \quad \eta = y$, and the motion of the medium in regions 2 and 3 of loading and unloading is described by the wave equation of the potential of the velocity φ we have the wave equation [1-4]

$$\mu^2 \frac{\partial^2\varphi}{\partial\xi^2} - \frac{\partial^2\varphi}{\partial\eta^2} = 0, \quad \left(\mu^2 = \frac{D^2}{C_P^2} - 1\right) \text{in plane deformation.}$$

We represent solutions in research areas in the form

$$\begin{aligned} \varphi_2(\xi,\eta) &= f_1(\xi - \mu\eta) + f_2(\xi + \mu\eta), \\ \varphi_3(\xi,\eta) &= f_3(\xi - \mu\eta) + f_4(\xi + \mu\eta), \end{aligned} \tag{1}$$

where $\varphi_2,\ \varphi_3 -$velocity potentials.

To find the unknown functions f_1 and f_2 i.e. to solve the problem in region 2, we have the conditions for the continuity of the velocities on the characteristic AB and the condition that at different horizontal levels $(\eta = const)$ the pressure of the medium in front of the reflected wave is equal to the pressure at the front of the incident wave. This means that the state of the medium in region 1 is on the unloading branches of the $P \sim \varepsilon$ diagram, and after the arrival of perturbations from the rigid boundary using the characteristic AB, in region 2 the pressure increases continuously to values determined by the points of intersection of the unloading and loading branches of the $P \sim \varepsilon$ diagram. Subsequently, under the action of the reflected plastic wave AD an abrupt increase in pressure occurs. This means that the reduced media in regions 2 and 3, according to the hydrostatic compressed, obeys Prandtal's scheme. A similar picture takes place in the rod theory with the difference that, in this case, the loading of the medium starts from the perturbed unloading region 1.

Thus, to solve the problem in domain 2 in the case of an exponential load

$f(\xi)=P_0 e^{-\gamma\xi}, \quad \gamma>0, \quad \xi\geq 0$ we have the conditions

on characteristic AB, i.e. when $\xi+\mu\eta=\dfrac{(1+\mu tg\alpha)}{tg\alpha}$

$$u_2=\frac{\partial\varphi_2}{\partial\xi}=-\frac{P_0}{\rho_0 D}\Psi_9(\xi,\eta), \tag{2}$$

$$\vartheta_2=\frac{\partial\varphi_2}{\partial\eta}=-\frac{P_0}{\rho_0 D}\Psi_{10}(\xi,\eta), \tag{3}$$

where

$$\Psi_9(\xi,\eta)=\left\{\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\gamma\xi}\left[e^{\lambda^{n+1}\gamma\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h-\xi\right)}-e^{-\lambda^{n+1}\gamma\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h-\xi\right)}\right]-e^{-\gamma\left(2\xi-\frac{(1+tg\alpha)}{tg\alpha}h\right)}\right\}$$

$$\Psi_{10}(\xi,\eta)=\left\{\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\gamma\left(\frac{2h}{tg\alpha}-\xi\right)}\left[e^{\lambda^{n+1}\gamma\mu tg\alpha\left(\frac{2h}{tg\alpha}-\xi\right)}-e^{\lambda^{n+1}\gamma\mu tg\alpha\left(\frac{2h}{tg\alpha}-\xi\right)}\right]+e^{\gamma(1-\mu tg\alpha)\left(\frac{2h}{tg\alpha}-\xi\right)}\right\}$$

in the section AE of the reflected wave front, i.e. when $\eta+\xi tg\alpha=2h$ [5-10]

$$P_2=-\rho_0 D u_2=P_0\Psi_{11}(\xi,\eta). \tag{4}$$

where

$$\Psi_{11}(\xi,\eta)=\left\{\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\gamma\xi}\left[e^{\lambda^{n+1}\gamma\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h-\xi\right)}+e^{-\lambda^{n+1}\gamma\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h-\xi\right)}\right]+e^{-\gamma\left(2\xi-\frac{(1+tg\alpha)}{tg\alpha}h\right)}\right\}\text{cm.}$$

Then, substituting (1) into (2), (4) taking into account (3), we obtain the expressions

$$f_1'(z)=-f_2'\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h\right)-\frac{P_0}{\rho_0 D}\Psi_{12}(\xi,\eta)-\frac{P_0}{\rho_0 D}e^{-\gamma z}, \tag{5}$$

$$f_2'(z)=-f_2'\left(\frac{1+\mu tg\alpha}{tg\alpha}h\right)+\frac{P_0}{\rho_0 D}\Psi_{13}(\xi,\eta)-\frac{P_0}{\rho_0 D}e^{-\gamma\left(\frac{(z-2\mu h)}{\lambda}-2\mu h\right)}\frac{P_0}{\rho_0 D}\Psi_{14}(\xi,\eta), \tag{6}$$

$$f_2'\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h\right)=\frac{P_0}{\rho_0 D}\sum_{n=1}^{\infty}\lambda^{-\lambda^{n+1}\frac{(1+\mu tg\alpha)}{tg\alpha}}. \tag{7}$$

where

$$\Psi_{12}(\xi,\eta)=\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha}h+z\right)}\left[e^{\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha}h-z\right)}-e^{-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha}h-z\right)}\right],$$

$$\Psi_{13}(\xi,\eta)=\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{z-2\mu h}{\lambda}+\frac{(1+\mu tg\alpha)}{tg\alpha}h\right)}\times\left[e^{\lambda^{n+1}\frac{\gamma}{2}\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h+1\mu h-\frac{(z-2\mu h)}{\lambda}\right)}-e^{-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{(1+\mu tg\alpha)}{tg\alpha}h+2\mu h-\frac{(z-2\mu h)}{\lambda}\right)}\right],$$

$$\Psi_{14}(\xi,\eta)=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\lambda^{n+1}e^{-\lambda^{n+1}\gamma\left(\frac{2h}{tg\alpha}+\frac{(z-2\mu h)}{(1-\mu tg\alpha)}\right)}\left[e^{\lambda^{n+1}\gamma\mu tg\alpha\left(\frac{2h}{tg\alpha}+\frac{(z-2\mu h)}{(1-\mu tg\alpha)}\right)}e^{-\lambda^{n+1}\gamma\mu tg\alpha\left(\frac{2h}{tg\alpha}+\frac{(z-2\mu h)}{(1-\mu tg\alpha)}\right)}\right]+\\+e^{\gamma(1-\mu tg\alpha)\left(\frac{2h}{tg\alpha}+\frac{(z-2\mu h)}{(1-\mu tg\alpha)}\right)}\end{array}\right\},$$

So, the solution to the problem when using (5) and (6) will finally be written in the form

$$u_2(\xi,\eta)=\frac{\partial\varphi_2}{\partial\xi}=-\frac{P_0}{\rho_0 D}\Psi_{15}(\xi,\eta), \qquad (8)$$

$$\vartheta_2(\xi,\eta)=\frac{\partial\varphi_2}{\partial\eta}=-\frac{P_0}{\rho_0 D}\Psi_{16}(\xi,\eta), \qquad (9)$$

where $\lambda=\dfrac{1-\mu tg\alpha}{1+\mu tg\alpha}$,

$$\Psi_{15}(\xi,\eta)=\left\{\sum_{n=0}^{\infty}2\lambda^{n+1}\left[sh\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha}h-\xi+\mu\eta\right)\right)\cdot e^{\left(-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha}h-\xi+\mu\eta\right)\right)}-\right.\right.$$

$$-sh\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+3\mu tg\alpha}{tg\alpha}+2\frac{\mu}{\lambda}\right)h-\frac{(\xi+\mu\eta)}{\lambda}\right)\cdot e^{\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+3\mu tg\alpha}{tg\alpha}+2\frac{\mu}{\lambda}\right)h-\frac{(\xi+\mu\eta)}{\lambda}\right)}+$$

$$\left.+\left(\lambda^{n+1}\gamma tg\alpha\left(\frac{2h}{tg\alpha}-\frac{\xi+\mu\eta-2\mu h}{1-\mu tg\alpha}\right)\right)\cdot e^{\left(\lambda^{n+1}\gamma tg\alpha\left(\frac{2h}{tg\alpha}-\frac{\xi+\mu\eta-2\mu h}{1-\mu tg\alpha}\right)\right)}\right]+$$

$$\left.+\left[e^{\gamma(\xi-\mu\eta)}+e^{-\gamma(1-\mu tg\alpha)\left(\frac{2h}{tg\alpha}-\frac{\xi+\mu\eta}{1-\mu tg\alpha}\right)}-e^{-\gamma\left(\frac{\xi+\mu\eta}{\lambda}-2\mu\right)\left(1+\frac{1}{\lambda}\right)}\right]\right\}$$

$$\Psi_{16}(\xi,\eta)=\left\{\sum_{n=0}^{\infty}2\lambda^{n+1}\left[sh\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+3\mu tg\alpha}{tg\alpha}+2\frac{\mu}{\eta}\right)p+\frac{\xi+\mu\eta}{\lambda}\right)\cdot e^{\left(-\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+3\mu tg\alpha}{tg\alpha}+2\frac{\mu}{\eta}\right)h\right)}-\right.\right.$$

$$\left.+sh\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{\xi+\mu\eta}{\lambda}-2\mu\right)\left(1+\frac{1}{\lambda}\right)\right)\cdot e^{\left(\lambda^{n+1}\frac{\gamma}{2}\left(\xi+\mu\eta+\frac{1+\mu tg\alpha}{1-\mu tg\alpha}h\right)\right)}+e^{\lambda^{n+1}\gamma\frac{(1+\mu tg\alpha)}{tg\alpha}}\right]+$$

$$\left.+\left[e^{\gamma(\xi-\mu\eta)}+e^{-\gamma\left(\frac{\xi+\mu\eta}{\lambda}-2\mu\right)\left(1+\frac{1}{\lambda}\right)h}-e^{-\gamma\left(\frac{2h}{tg\alpha}-\frac{\xi+\mu\eta-2\mu}{1-\mu tg\alpha}\right)}\right]\right\}$$

Now let's start solving the problem in region 3. For this, we have the following conditions: at the front of the reflected wave *AE*, i.e. at $\eta+\xi tg\alpha=2h$ [5,6-10],

$$\rho_0 a\left(\vartheta_{n3}-\vartheta_{n2}\right)=P_3-P_2, \qquad \vartheta_{r3}=\vartheta_{r2}, \qquad (10)$$

on a rigid boundary at $\eta=h, \quad \xi_a\le\xi\le\xi_c$

$$\frac{\partial\varphi_3}{\partial\eta}=0. \qquad (11)$$

Given that $\vartheta_n = -u\sin\alpha - \vartheta\cos\alpha, \quad \vartheta_\tau = u\cos\alpha - \vartheta\sin\alpha, \quad a = D\sin\alpha,$ (12)

from (10) we obtain

$$\left(\vartheta_3 - \vartheta_2\right)tg\alpha \cdot = u_3 - u_2 . \qquad (13)$$

From the second equation in (1), taking into account (10) and (11) with respect to the function f_3 and f_4 we obtain a system of equations in the form

$$f_3'(z) = f_4'(z + 2\mu h), \qquad (14)$$

$$f_4'(z) + \lambda f_4'(z + 2\mu h) = G(z), \qquad (15)$$

where

$$G(z) = -\frac{P_0}{\rho_0 D}\left\{\sum_{n=0}^{\infty} 2\lambda^{n+1}\left[sh\left(\lambda^{n+1}\frac{\gamma}{2}\left(\frac{1+\mu tg\alpha}{tg\alpha} + 2\mu\right)h - z\right) e^{-\lambda^{n+1}\frac{\gamma}{2}\left(z + \frac{1+\mu tg\alpha}{tg\alpha}h\right)}(1-\lambda) + \right.\right.$$

$$\left. + \lambda sh\left(\lambda^{n+1}\gamma\mu tg\alpha\left(\frac{2h}{tg\alpha} - \frac{z}{1+\mu tg\alpha}\right)\right) e^{\lambda^{n+1}\gamma\left(\frac{2h}{tg\alpha} - \frac{z}{1+\mu tg\alpha}\right)}\right] + e^{-\gamma(z - 2\mu h)}(1-\lambda) +$$

$$\left. + \lambda e^{-\gamma(1-\mu tg\alpha)\frac{2h}{tg\alpha}} e^{\gamma\lambda z} + (1-\lambda)\sum_{n=0}^{\infty}\lambda^{n+1} e^{-\lambda^{n+1}\gamma\frac{(1+\mu tg\alpha)}{tg\alpha}h} \right\}$$

Solving equation (15), by the method of successive approximations, it is easy to obtain the formula

$$f_4'(z) = G(z) + \sum_{n=0}^{\infty}(-\lambda)^m G\left[\lambda^m z + 2\mu h\frac{(\lambda^m - 1)}{(\lambda - 1)}\right]. \qquad (16)$$

Thus, for region 3 we have a solution to the problem in the form

$$u_3(\xi,\eta) = G(\xi - \mu\eta + 2\mu h) + G(\xi + \mu\eta)\Psi_{17}(\xi,\eta), \qquad (17)$$

$$\vartheta_3(\xi,\eta) = \mu\left[G(\xi + \mu\eta) - G(\xi - \mu\eta + 2\mu h) + \Psi_{18}(\xi,\eta)\right], \qquad (18)$$

where

$$\Phi_{17}(x,t)=\left\{\begin{aligned}
&U_2(x-l)\left[1-\exp\left(-\frac{bE_0t}{a\rho}\right)\right]\left[1-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)-C\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)\right]\times\\
&\times\left[\frac{1}{2}-\frac{1}{2}\exp\left(-\frac{bE_0t}{2a\rho}\right)\right]+\sqrt[4]{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{x+l}{8\sqrt{2\pi}}\sum_{n=0}^{\infty}\left[F_1(n,t,x+l)+F_2(n,t,x+l)\right]\times\\
&\times\frac{1}{n+1}\left[1+(-1)^n\exp\left(-\frac{bE_0t}{a\rho}\right)\right]-\frac{a^3\rho}{4bE_0}\sqrt{\frac{\rho}{IE}}\left[S\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)-C\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)\right]\times\\
&\times\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-4\exp\left(-\frac{bE_0t}{2a\rho}\right)-\frac{bE_0t}{2a\rho}+3\right]+\sqrt[4]{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{x+l}{4\sqrt{2\pi}}\times\\
&\times\sum_{n=0}^{\infty}\left[F_2(n,t,x+l)-F_1(n,t,x+l)\right]\left[2+\left((-1)^n\exp\left(-\frac{bE_0t}{a\rho}\right)+\frac{bE_0t}{a\rho}-3\right)\frac{1}{n+1}\right]-\\
&-U_1(l-x)\left[1-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)-C\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)\right]\times\\
&\times\left[\frac{1}{2}-\frac{1}{2}\exp\left(-\frac{bE_0t}{2a\rho}\right)\right]-\sqrt[4]{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{|x-l|}{8\sqrt{2\pi}}\sum_{n=0}^{\infty}\left[F_1(n,t,|x-l|)+F_2(n,t,|x-l|)\right]\times\\
&\times\frac{1}{n+1}\left[1+(-1)^n\exp\left(-\frac{bE_0t}{a\rho}\right)\right]+\frac{a^3\rho}{4bE_0}\sqrt{\frac{\rho}{IE}}\left[S\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)-C\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)\right]\times\\
&\times\left[\begin{aligned}&\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-4\exp\left(-\frac{bE_0t}{2a\rho}\right)-\frac{bE_0t}{a\rho}+3\right]+\sqrt[4]{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{|x-l|}{4\sqrt{2\pi}}\times\\ &\times\sum_{n=0}^{\infty}\left[F_2(n,t,|x-l|)-F_1(n,t,|x-l|)\right]\left[2+\left((-1)^n\exp\left(-\frac{bE_0t}{a\rho}\right)+\frac{bE_0t}{a\rho}-3\right)\frac{1}{n+1}\right]\end{aligned}\right]+\\
&+\frac{a^3\rho}{bE_0}\sqrt{\frac{\rho}{IE}}\left[\begin{aligned}&\left[C\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)\right]\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-1+\frac{bE_0t}{2a\rho}\right]+\\ &+\sqrt{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{x+l}{4\sqrt{2\pi}}\sum_{n=0}^{\infty}\left[F_2(n,t,x+l)-F_1(n,t,x+l)\right]\end{aligned}\right]+\\
&+\left[1+\left(\left(\frac{bE_0t}{2a\rho}-1\right)\right)\frac{1}{n+1}\right]+U_1(l-x)\times\\
&\times\left[\begin{aligned}&\left[C\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x-l)^2}{4t}\right)\right]\left(\exp\left(-\frac{bE_0t}{2a\rho}\right)-1+\frac{bE_0t}{2a\rho}\right)+\\ &+\sqrt[4]{\frac{\rho}{IE}}\frac{bE_0}{a\rho}\frac{|x-l|}{4\sqrt{2\pi}}\sum_{n=0}^{\infty}\left[F_2(n,t,|x-l|)-F_1(n,t,|x-l|)\right]\left[1+\left(\left(\frac{bE_0t}{2a\rho}-1\right)\right)\frac{1}{n+1}\right]\end{aligned}\right]
\end{aligned}\right\},$$

$$
\Phi_{18}(x,t)=\left\{\begin{aligned}
&U_2(x-l)\frac{a\rho}{bE_0}\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-1+\frac{bE_0t}{a\rho}\right]-\cos\left(\sqrt{\frac{\rho}{IE}}\frac{az}{2}\right)\times\\
&\times\left[\begin{aligned}
&\frac{a\rho}{bE_0}\left[1-C\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)\right]\times\\
&\times\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-1+\frac{bE_0t}{a\rho}\right]+\sqrt[4]{\frac{\rho}{IE}}\frac{x+l}{8\sqrt{2\pi}}\times\\
&\times\sum_{n=0}^{\infty}\left[F_1(n,t,x+l)+F_2(n,t,x+l)\right]\times\\
&\times\left[\frac{1}{n+1}\left(1-(-1)^n\exp\left(-\frac{bE_0t}{a\rho}\right)-\frac{bE_0t}{a\rho}\right)-2\right]
\end{aligned}\right]-\sin\left(\sqrt{\frac{\rho}{JE}}\frac{az}{2}\right)\times\\
&\times\left[\begin{aligned}
&\frac{a\rho}{2bE_0}\left[C\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)-S\left(\sqrt{\frac{\rho}{IE}}\frac{(x+l)^2}{4t}\right)\right]\\
&\left[\exp\left(-\frac{bE_0t}{a\rho}\right)-1+\frac{bE_0t}{a\rho}\right]+\sqrt{\frac{\rho}{IE}}\frac{x+l}{8\sqrt{2\pi}}\\
&\sum_{n=0}^{\infty}\left[F_2(n,t,x+l)-F_1(n,t,x+l)\right]\\
&\left[2+\frac{1}{n+1}\left((-1)^{n+1}\exp\left(-\frac{bE_0t}{a\rho}\right)+\frac{bE_0t}{a\rho}-1\right)\right]
\end{aligned}\right]+
\end{aligned}\right\}\times
$$

The pressure in the region is determined by the formula

$$
P=-\rho_0 Du_i,\quad (i=2,3). \tag{19}
$$

Conclusion. An analytical solution to the problem of the propagation of a plastic wave in a half-space is constructed in the case when the relationship between pressure and volumetric deformation during loading and unloading is linear, but different. Based on the analysis of the calculation results, it is shown that if the moving load acting on the boundary of the half-space has a monotonically decreasing profile, then in the disturbance region, the medium is unloaded and an oblique compression wave is obtained by the load-unloading wave. The pressure of the medium against the background of this wave, depending on the depth of the half-space, decreases slowly than on the free surface. In the case when the dependence between P and during loading of the medium is assumed to be nonlinear and shock, which corresponds to the propagation of a shock wave in the medium, the pressure in the perturbed region, in comparison with the linear case, is somewhat overestimated.

References

[1].Rakhmatulin Kh.A., Demyanov Yu.A. Durability under intense short-term loads. Moscow, Logos. 2009 –P. 512 P.

[2].A. Aydosov, G. A. Aidosov, E.S. Temirbekov, S.N. Toybaev. Mathematical modeling of shock load propagation in continuous deformable media and the interaction of two deformable media under dynamic moving loads - Almaty, 2015, - 208 P. - ISBN 928-601-263-327-6.

[3].A. Aydosov, E. S. Temirbekov. Impact theory - Textbook. - Almaty: ATU, 2015. - 64 P.- ISBN 978-601-263-323-8.

[4]. A. Aydosov, G. A. Aidosov, M.N. Kalimoldaev, S.N. Toybaev, Mathematical modeling of the interaction of a beam (plates, plates, strips) with a deformable base under dynamic loads - Monograph / - Almaty, 2015, - 208 P.

[5].Aidosov G.A., Aidosov A.A., Toybaev S.N. Modeling the interaction of a beam (plates, slabs, strips) of variable thickness, lying on a non-uniform foundation. Formulation of the problem. Bulletin of K.I. Satpayev KazNTU, №4, Almaty: 2009. – P. 41–45. 0,25

[6].Aidosov A.A., Aidosov G.A., Toybaev S.N. Modeling the interaction of a beam (plates, slabs, strips) of variable thickness, lying on an inhomogeneous foundation. General equations. Bulletin of Al-Farabi KazNU, №1 (60), Almaty: 2009. – P. 48 53.

[7].Aidosov A.A., Aidosov G.A. Toybaev S.N. The main conclusions of modeling the propagation of blast waves in a multilayer inhomogeneous half-space Science news of Kazakhstan Scientific and technical collection. Issue 2 (101), Almaty: 2009. – P. 56- 60.

[8].Aidosov, A., Mamadaliev, N., Khakimov, U. Effect of a mobile load on a nonlinearly compressed strip with a rigid foundation (Article) - Journal of Applied Mechanics and Technical PhysicsVolume 27, Issue 3, May 1986, Pages 441-445 - Scopus – **36** pr.

[9].Aidosov A.A., Aidosov G.A., Toybaev S.N. Modeling the interaction of a beam (plates, slabs, strips) of variable thickness, lying on a non-uniform foundation. Problem statement // Bulletin of K.I. Satpayev KazNTU. – Almaty, 2009. - №4. – P.41–46.

[10].Aidosov A.A., Aidosov G.A., Toybaev S.N. Modeling the interaction of a beam (plates, slabs, strips) of variable thickness, lying on a non-uniform foundation. General solution to the problem// Bulletin of K.I. Satpayev KazNTU. Almaty, 2009. - №5. – P.103 – 108.