Ethnomathematical problems of Mongolian-speaking peoples in education

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Abstract. The article examines the ethnomathematical problems known among the Mongols, Buryats and Kalmyks of Russia, in comparison and comparison with the problems of other peoples and ways of solving them. From the content of the national tasks it becomes clear that they are of a practical nature, put forward by the very life of the nomads, their way of life. Tasks by methods of solution can be divided into finding parts of a whole, dividing into proportional parts, percentages, parity properties of numbers, arithmetic and geometric progressions, equations of the first and second degree, indefinite equations, problems with mixtures, etc. The use of ethnomathematical problems in school education allows students to become familiar with the history of mathematical science. An analysis of their content shows the similarity with the tasks of different peoples, confirms the similarity of the ethnomathematics of the Mongol-speaking peoples - the Zurkhai with the mathematics of China, India, the countries of Islam, Ancient Greece.

Keywords: ethnomathematics; ethnomathematics of Mongolian-speaking peoples - zurkhai; ethnomathematical problems of the Mongol-speaking peoples, methods of solving them and comparing them with the problems of other peoples.

Modern mathematical education is based on the principles of ethnocultural identification and integration of the individual into the world community. In the works of Russian scientists N.I. Merlinoy N.I., A.I. Petrova, V.M. Berkutova, M.D. Dyachkovskaya et al. Finds a place for the study of the mathematical heritage of the peoples of Russia, emphasizes the need to develop content and use it in education [3, 6]. The issues of the emergence, preservation and application of the initial mathematical concepts of different peoples in teaching are considered in the works of Ubiratan d'Ambrosio, A. Bishop, K. Zaslavsky, M. Alberti, B.L. Yashin and others, who laid the foundation for a new scientific direction - ethnomathematics [1, 7].

However, the content of the ethnomathematics of the Mongol-speaking peoples, called *zurkhai*, has not been sufficiently studied, the accumulation and systematization of the accumulated information continues. In this paper, we will consider some folklore forms that are modeled into mathematical problems using the concepts of numbers, figures, units of measurement, relations, equations, inequalities, progressions, percentages. We call such folklore forms ethnomathematical folklore [4, 5]. The main work revealing the history of the development of ethnomathematics of the Mongol-speaking peoples of the *zurkhai* is the work of B. Batzhargal "Ertniy Mongolian mathematician" ("Early mathematics of the Mongols"). It reflects the results of the analysis of many Tibetan, Sanskrit, Chinese, Mongolian manuscripts, woodcuts and Mongolian folklore texts collected by him [2].

From the ethnomathematical folklore of the Mongol-speaking peoples, we will try to single out folk solutions, correlating them with the solutions known among other peoples.

1. Method of adding missing

Problem 1 (Mongolian). One old man had three sons. Once he wrote a will: "After my death, let the eldest son get half of my herd of camels, the middle one - one third of them, and the youngest one - ninth." After the father left for another world, the sons could not divide the 17 camels into either 2, 3 or 9. Then they asked the wise old man for help, who divided the camels according to the stipulated condition. How many camels did each son get?

We note that the camel is one of the five sacred animals of the Mongol-speaking peoples, on which from ancient times they transported goods along the Great Silk and Tea Routes. Similar tasks are encountered in different nations of the world. In the solution known among the Central Asian peoples, the sage adds an donkey or mule to the herd, and in the Mongolian folk problem, the sage adds his camel when dividing, and then takes it back.

Solution The number of camels must be divisible by 2, 3, and 9, LCM (2, 3, 9) = 18. However, the sum of the shares

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{9}=\frac{17}{18}$$

differs from one. If we add $\frac{1}{18}$, that is, add one camel, then the herd is easy to divide according to the father's will. One son will receive 9 camels, the second 6, the third 2. Answer: 9, 6 and 2 camels.

From a methodological point of view, this task leads students to understand the concept of the least common multiple, the concept of the whole. It also deals with fractions (unit fractions). Parts and shares have been recognized by people since time immemorial. In Mongolian languages, they are called *khuv* (among the Mongols), *khubi* (among the Buryats). The way of reading fractions is different from the modern one, in ethnomathematics *zurkhai* is first called the denominator and then the numerator. So, the fraction $\frac{12}{18}$ is read as *eighteenths of twelve*.

2. A way to solve a problem from the end

Problem 2 (Buryat). The father and mother divided the flock of sheep among all the sons according to folk tradition. The youngest son was given half of the 256 heads of the flock, since he was supposed to provide his parents with a decent old age, and each of the remaining brothers got half as much as the previous one. As a result, the eldest son received 2 sheep. How many sons were there in this family?

S o l u t i o n .

2 + 4 + 8 + 16 + 32 + 64 + 128 = 254. At the same time, two sheep remain with their parents according to folk tradition. A n s w e r : 7 sons.

This problem is similar to the Indian problem from the Bakhshali manuscript (III century): "*B* gives twice as much as A; C - three times more than B; D - four times more than C. All together give 132. How much did A give?" The manuscript gives the following solution: "Let the unknown be equal to 1, then A = 1, B = 2, C = 6, D = 24, the sum is 33. Now let's divide 132 by 33. This will be the desired value." The Hindus acted in the reverse order and called this method of decision "the rule of circulation." The Buryat problem of a flock of sheep was also solved from the end by the inversion method, that is, the Mongolian peoples used a method similar to the Indian "rule of circulation".

3. Enumeration method

Problem 3 (Buryat). A flock of birds flew up to the trees. If each of them sits on one tree, then one bird will be superfluous. If the birds sit on trees in two, then one tree will be superfluous. How many birds were there and how many trees?

Here are similar problems encountered by other peoples.

Bulgarian folk problem. Donkeys grazed in the clearing. Several guys approached them. "Get on the donkey, one at a time!" - commanded the eldest of the guys. One boy was left without a donkey. “Get off! Get 2 on the donkey, ”the elder commanded again. One donkey was left without a boy. How many donkeys and how many children were there in the clearing?

Problem from the collection of fairy tales "1001 nights" (night 458). A flock of pigeons flew up to a tall tree. Some of the pigeons sat on the branches, and the other settled under the tree. The pigeons sitting on the branches say to those located below: "If one of you flew towards us, then you would become three times less than us together, and if one of us flew to you, then you and I would become equal." How many pigeons were sitting on the branches and how many under the tree?

Euclid's problem (III century BC). A mule and a donkey walked along the road with sacks under the pack. The donkey groaned pitifully, crushed by an overwhelming burden. This mule, who noticed, turned to his companion with a speech: "Well, old man, are you whining and crying like a girl? I would bear twice as much as you, if you gave me one measure. If you only took one from me, then we would be equal." How much each of them carried, oh, geometer, tell us this.

As you can see, in these tasks the plot is similar, known to many peoples. We believe that the problem was solved by the brute force method, since brute force is one of the most ancient methods found among all peoples at the early stages of mathematics development.

S o l u t i o n .

Method 1.Choosing the first condition of the problem: there are 1 more birds than trees, consider pairs of numbers expressing the number of both: 2 and 1, 3 and 2, 4 and 3. Assuming that there are 4 birds, we are convinced that the second condition of the problem performed. This means that there are four birds and three trees. In the other two cases, the second condition is not met.

Method 2. Let's try to find the required numbers, given that they must be integers. Let's say there were two birds. According to the first condition, we plant the birds one by one on the trees. Since in this case one bird will remain superfluous, the tree will be one. But then the second condition is not met: if two birds sit down one tree at a time, there will be no extra trees. This means that there are not two birds. Similarly, consider the case when there are three birds; it also does not satisfy the condition of the problem. Assuming that there are four birds, we make sure that both conditions of the problem are met. A n s w e r : 4 birds, 3 trees.

Solution does not require special mathematical knowledge, you can perform a full possible enumeration of options. However, the task has great didactic value: by looking over possible solutions, students develop intuition, skills of oral counting, experiment, observe, and draw general conclusions based on particular results.

Folklore tasks of Mongolian-speaking peoples are used in the classroom to create motivation to study a new topic, stages of consolidation and generalization of what has been learned. They can be categorized according to some topics of the school curriculum. Here are some examples.

Topic "Actions with natural numbers"

Problem 4 (Buryat). The man became a father at the age of 17, 19 and 21. If the youngest of the sons is now 3 years old, how old are the eldest sons? Answer: 5 years and 7 years.

The next problem is interesting because the answer is a multi-digit number. This suggests that the Mongols, Buryats, Kalmyks operated with large numbers and gave them names.

Problem 5 (Mongolian). Old man Erentai had 99 children, each of them had 9 children. Each of the 9 children had 7 children, each of the 7 children had 3 children. How many were there in total?

S o l u t i o n .

1 + 99 + 99 × 9 + 99 × 9 × 7 + 99 × 9 × 7 × 3 = 1 + 99 + 891 + 6237 + 18 711 = 25 939.
A n s w e r . 25 939 people.

Topic "Fractions"

Problem 6 (Mongolian). 24 dan of cereals were given to help four poor women. They were an orphan, a childless woman, a widow and a pregnant woman. They were given $\frac{4}{25}$, $\frac{1}{5}$, $\frac{7}{25}$ and $\frac{9}{25}$. respectively. The question is, how much grain did each of them get (1 dan = 10 duu, 60 kg, 1 duu = 10 shin)? A n s w e r : orphan –3 dan 8 duu 4 shin ≈ 230.4 kg; childless woman – 4 dan 8 duu ≈ 288 kg; widow– 6 dan 7 duu 2 shin ≈ 403.2 kg; pregnant– 8 dan 6 duu 4 shin ≈ 518.4 kg.

This problem was solved using the calculating devices *sampin* and *zurhain sambar* (counting board), which were in use until the 50s of the last century. The *sampin* resembled Chinese and Japanese abacus and was six-point (one bone was separated by a septum from the rest). The existence of the sampin, along with a special count of up to ten among the Mongols, confirms the early existence of a mixed number system among the Mongol-speaking peoples: five-decimal. *Zurhain sambar* is a small board about 25 × 40 cm in size, which was coated with sheep's fat, sprinkled with fine sand or dust and wrote on it with a sharpened stick.

Topic "Systems of equations"

In the Middle Ages, the rich mathematical tradition of the Chinese people became available to Mongolian scholars. Obviously, this is why tasks that were solved using the "rule of false position", "the rule of two false positions" penetrated into the people's environment. Similar problems are considered in the VII-VIII books of the Chinese treatise "Mathematics in nine books", where an algorithm for their solution is given.

Problem 7 (Mongolian). There were 49 birds and sheep in the hedge. They have 100 legs in total. How many birds and how many sheep were there in the hedge? A n s w e r : 48 birds, 1 sheep.

The problem is solved using the system of equations

$$\begin{cases} x + y = 49; \\ 4x + 2y = 100; \end{cases}$$

where x – number of sheep, y –number of birds.

Topic "Divisibility. Even and odd numbers"

Problem 8 (Buryat). One horse herder can harness five, group three, or disband two. How many horses can there be in a herd? A n s w e r : 30*n*, n = 1, 2, 3, ….

Problem 9 (Mongolian). In the old days, they say, an old man was on a journey with a young traveler, who guarded the camels every night. Once a young man asked an old man: "How can I be if I want to sleep at night?" “You tie our 70 camels to 7 pegs so that an odd number of camels is tied to each peg!” - answered the old man. The young man thought and thought and did not notice at all how dawn was breaking. What is the secret of the old man's advice? A n s w e r : the problem has no solution.

Topic "Arithmetic and Geometric Progressions"

An analysis of the solution of folk problems shows that the Mongol-speaking peoples knew the concepts of the degree of number, arithmetic and geometric progression. Let's take the problem of lamps as an example.

Problem 10 (Mongolian). On the Buddhist seven-step step - *subargan*, in the shape of a pyramid, 381 lamps are placed so that the number of lamps on the steps doubles each time, starting from top to bottom. How many icon lamps are there on the top of the *subargan* and how many on each step? A n s w e r : at the top there are 3 icon lamps, and on the rest of the steps6, 12, 24, 48, 96, 192.

Problems for extracurricular activities

Folklore tasks of different peoples are traditionally used in the organization of extracurricular work with students. Logical problems, game problems, transfusion problems, etc., occupy a prominent place in the mathematical collection of folklore problems of the Mongol-speaking peoples. Let us give examples.

1. Logical tasks

Problem 11 (Buryat). The traveler leads three camels with their luggage. One camel bites the others, the second spits chewing gum at the owner, and the third eats the baggage. How can a traveler lead the camels so that no one is hurt and the luggage is safe? A n s w e r : the traveler must sit on a camel that bites the others, lead a camel that can eat the load, shifting the load onto it, and tie the spitting camel to it.

Problem 12 (Mongolian). Once an old man and an old woman gave their three sons a gold coin and sent them to buy something of value. The eldest son bought a mirror with his gold coin, in which one could see the invisible, the middle son - a flying carpet, and the youngest - a cure for any disease. The eldest son looked in his mirror and saw a sick beauty. The brothers sat down on a flying carpet and flew to her. Then the younger of the brothers cured the beauty with his miraculous medicine. One of the brothers married her. Which brother married a beautiful woman? A n s w e r : the youngest of the brothers married the beauty, who bought medicines for all diseases with a gold coin; according to the folk tradition of the Buryats, he is supposed to protect the calm old age of his parents.

2. Game-problems.

Problem 13 (Mongolian). *Game "Guess who has the ring."* 11 people play, one of whom plays the game, the second guesses, the rest of the participants hide the ring. Guessing: “Guys! Have one of you put this ring on your finger. I will find out which of you has, which hand, which finger and its phalanx it is worn. Let's agree that: a) we will assign numbers from 1 to 9 to each of you; b) the right hand will be numbered 1, and the left hand will be numbered 2; c) fingers, starting from the thumb, will be numbered from 1 to 5; d) phalanges of fingers, starting from the tip, will be numbered from 1 to 3". Then the guesser leaves the room. While he is absent, the presenter assigns numbers to the participants, puts a ring on someone's finger and asks everyone to hide their hands. For example, the presenter puts a ring on the first phalanx of the ring finger of the fifth participant's right hand and invites the guesser to enter the room.

The guesser enters and invites the participants to correctly perform the following calculations: 1) multiply the number of the participant in the game with the ring hidden by 2 and add 1 to the product; 2) multiply the received amount by 5 and add the hand number to the product; 3) multiply the sum by 2 and add 1 to the product; 4) multiply the sum by 5 and add the number of the finger to the product; 5) multiply the sum by 2 and add 1 to the product; 6) multiply the sum by 5 and add the number of the phalanx of the finger to the product. Then the guesser asks the answer.

In this case, the result will be 5796. From the result 5796, the leader subtracts 555 orally and receives the answer 5241. According to this answer, he finds out who the ring was hidden from: 5 - the player's number, 2 - the number of his hand, 4 - the number of the finger, 1 - the number phalanges of the finger.

Task for students. Prove the correctness of the reasoning, denoting with the letter *a* the player's number, *b* - the hand number, c - the finger number, d - the phalanx number of the finger.

S o l u t i o n .

$$((((((2a + 1) \times 5) + b) \times 2 + 1) \times 5 + c) + 1) \times 5 + d = 1000a + 100b + 10c + d + 555= abcd + 555.$$

Problem 14 (Buryat). *Game "Guess the number".* Come up with a number, add it to it. I give you the number 10, add it to the total as well. Divide the resulting number by 2, then subtract the intended number. I guessed how many turned out. It turned out 5, right? What's the secret?

3. Transfusion problems.

In summer, families of cattle breeders have an abundance of dairy products, when they have to pour milk, butter, sour cream from one vessel to another. This is probably why numerous tasks of "transfusion" arose, similar to the tasks of other peoples. Here's one example.

Problem 15 (Mongolian). How should two cattle breeders divide equally 10 ghee if they have vessels for 7 and 3 ghee? (1 ghee - a measure of weight equal to 600 g)

Topic "Indefinite Equations"

A rare problem for indefinite equations was written by the author in the Buryat village of Shibertui from D. Mitypov.

Problem 16 (Buryat). The father ordered his son to buy 100 head of cattle for 5 rubles. At the same time, a horse cost 50 kop., a cow - 10 kop., a sheep - 1 kop. The son fulfilled his father's order. How many horses, cows, sheep did he buy?

S o l u t i o n . Let's compose a system of two equations with three unknowns:

$$\begin{cases} 50\ x + 10y + z = 500, \\ x + y + z = 100, \end{cases}$$

where x – number of horses, y – number of cows, z – the number of sheep (all natural numbers). Subtract the second equation from the first and express y in terms of x, we get y= (400- $-$ 49x) : 9. The problem has only one solution: x = 1, y = 39, z = 60. A n s w e r : 1 horse, 39 cows, 60 sheep.

Problem 17 (Mongolian). For 31 heads of foals, calves, camels were given 67 ghee feed. Of these, for one foal - 1 ghee, for one calf - 2 ghee, for one camel - 3 ghee. How many foals, calves, camels get food? A n s w e r :3 foals, 20 calves, 8 camels.

The problems mentioned are similar to the well-known Chinese problem of buying birds.

Zhang Qiujian's problem (V century). 1 rooster costs 5 qian*, 1 chicken costs 3 qian, 3 chickens cost 1 qian. In total, 100 birds were bought for 100 qiens. The question is, how many cocks, chickens, and chickens were there? (Qian- Chinese currency)

It is known that scientists from different countries have dealt with systems of equations in which the number of unknowns exceeds the number of equations. This makes it possible to inform students of interesting information from the history of mathematics of other peoples: in honor of the Alexandrian mathematician Diophantus (III century), such equations were called Diophantine. Scientists of India (V-XII centuries) were engaged in solving indefinite equations in integers. Such problems are considered in the VII-VIII books of the Chinese treatise "Mathematics in nine books", they are solved by the "excess-lack" method. In particular, the latter problems are analogous to Zhang Qiujian's well-known Chinese problem of buying birds:

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The examples considered show that the Mongol-speaking peoples had a large supply of original mathematical problems, for the solution of which a sharp mind, resourcefulness, and ingenuity were required. From the content of the folklore tasks it becomes clear that they were of a practical nature, were put forward by the very life of the nomads, their way of life [4], [5].

In the XII-XIII century, the ethnomathematics of the Mongol-speaking peoples of the *zurkhai* reached significant development. Researches of scientists have shown that at that time it was closely associated with the mathematics of India, China, Tibet and the countries of Central Asia, this connection was especially strengthened during the spread of Buddhism and the existence of a single state in the Middle Ages. The fact of mutual enrichment of the mathematical culture of the peoples of Central Asia is confirmed by folk problems created in different eras, in different places of residence of peoples.

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