Research of transport processes in information networks

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ABSTRACT

The report is devoted to research on the capacity of information networks and related stochastic fluctuations and bursts. In open systems, the exchange of energy and information with surrounding bodies, due to their complexity, generates the formation of various structures. This process of creating structures is especially relevant when it comes to systems with a fractal structure. Analysis of such processes should be carried out in terms of fractional geometry. The dynamics of such processes are characterized by such effects as memory, complex spatial mixing processes and self-organization. The use of fractional dynamics methods opens up new possibilities for solving problems of forecasting and decision making in complex systems.

Keywords:

Stochastic processes, entropy of Tsallis, branching processes, percolation, Deep Learning.

1. INTRODUCTION

It is known that during the operation of complex transport systems (networks) both the capacity of the network (channels) and the demand for traffic are subject to stochastic fluctuations and bursts (Levy flight) [1-5]. These random fluctuations are the main sources of uncertainty of the transit time, and a result, losses information (technological losses).

It is important to note that the heterogeneity of stochastic processes and the asymmetry of claims causes uncertainty, different from the traditional one. In the context of the coordinate of emerging problem, for the purpose of traffic assignment, the model of entropy of Tsallis is proposed that allows tracking coherent processes.

It is known that stochastic transport processes represent a generalization of the diffusion process, which is expressed in the transition from the usual root dependence to the ratio [1]:

$$\left\langle r^2 \right\rangle \propto t^{2/z}, \tag{1}$$

characterized by dynamic exponents $z \neq 2$ (here r - the coordinate of the wandering particle, t - time).

When sub diffusion, the presence of traps leads to a divergence of the average waiting time for jumps $\langle t \rangle = \infty$, so that the latter acquire a discrete character in space and the transport process slows down ($z > 2$).

Its acceleration in the process of super diffusion of levels is due to the fact that the particle at discrete instants of time performs jumps of arbitrary length, characterized divergent mean square displacement $\langle x^2 \rangle = \infty$ [1].

2. Mathematical Model of Fractional Traffic Levy Motion

α - stable Levy motion, $L_{\alpha,H}(t)$. The in terms of the Riemann-Liouville operator, we have [2, 3]:

$$L_{\alpha,H}(t) = \frac{1}{\Gamma\left(H + \frac{1}{2}\right)} \int_0^t dL_\alpha(\tau)(t-\tau)^{H-1/2}, \qquad (2)$$

where $L_\alpha(t)$ is the ordinary symmetric α - stable Levy Motion (oLm), and $\Gamma(\cdot)$ denotes the gamma function, H - Hurst parameter.

From a mathematical model of fractional traffic Levy will be expressed as [2, 3]:

$$\widetilde{A}(t) = mt + (\overline{\sigma} m)^{1/\alpha} L_{\alpha,H}(t), \qquad (3)$$

where $m > 0$ is the mean input rate, $\overline{\sigma}$ is the scale factor, and $L_{\alpha,H}(t)$ is the fLm process defined by (2).

The model has four parameters $m, \alpha, \overline{\sigma}$ and H with the following interpretations [2, 3]:

- $m > 0$ is the mean constant input rate
- $\alpha \in (1,2]$ measures the “thickness” of the tails of the stable distribution
- $\overline{\sigma} > 0$ is the scaling parameter that can be seen as the dispersion around the mean of the traffic
- $H \in \frac{1}{2}\frac{1}{\alpha}, \frac{3}{2}\frac{1}{\alpha}$ is the Hurst parameter (index of self-similarity)

Based on the mathematical model of traffic (3) transport models of the types: Branching processes as a particle branching and fractional Brownian motion (fBm).

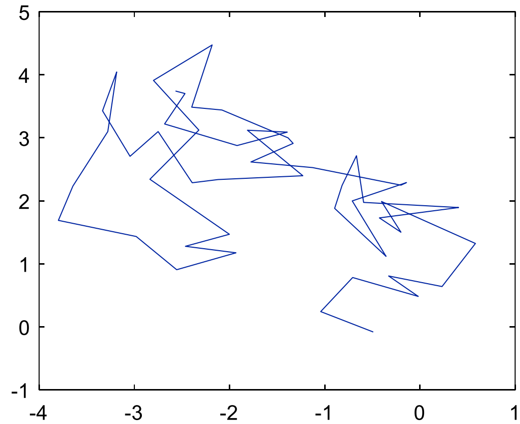

Figure 1. Levy motion.

2.1 Branching processes as a particle branching

In this section deals with general Bienayme-Galton-Watson processes describing branching particle systems in the discrete time setting [6, 7]. We denote by $Zn(A)$ the number of n-th generation particles whose types belong to $A \in \xi$. The same generation particles are assumed to produce offspring to a random algorithm.

A key characteristic of the multi-type reproduction law is the expectation kernel [4]:

$$M(x,A) := E_x Z_1(A),\ \ x \in A,\ \ A \in \varepsilon \ , \tag{4}$$

where the operator E_x is indexed by type x of the ancestral particle.

Here LF-processes, branching particle systems are characterized by general linear-fractional (LF) distributions.

It is assumed that the type of the desired genus x, the total number of offspring $Z1 :\approx Z1(E)$ follows linear-fractional distribution [6]:

$$E_x s^{Z1} = P_0(x) + (1 - P_0(x)) \frac{s}{1 + m - ms}\ , \tag{5}$$

where $m \in (0,\infty)$.

If $P_0(x) = 1 - k(x,E)$, where k - is kernel, the ancestral particle has no offspring, and with probability $1 - P_0(x)$, it produces a shifted-geometric number of offspring [6]:

$$E_x\left(s^{Z_p} \middle| Z_p > 0\right) = \frac{s}{1 + m - ms}\ , \tag{6}$$

where parameter m is independent of x.

Then on the basis of [7] the model of fractional traffic Bienayme-Galton Watson processes will have the form:

$$\overline{A}(t) = (et + \overline{\sigma} l) E_x s^{Z_1}, \tag{7}$$

where $l > 0$, $\overline{\sigma}$ - scale factor.

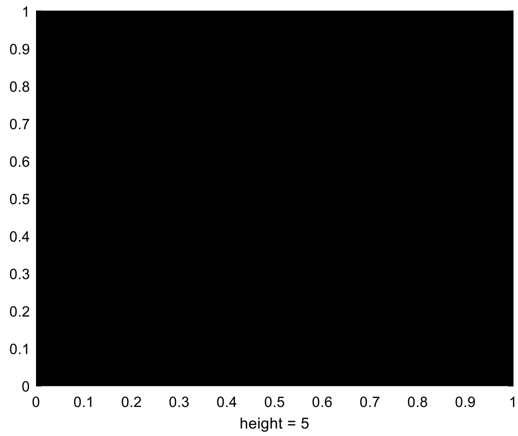

Figure 2. Branching process.

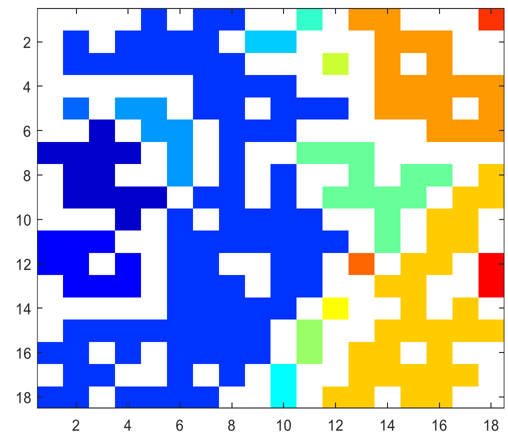

Figure 3. Percolation lattice.

2.2 Fractional Brownian Motion (fBM)

Fractional Brownian motion is defined by its stochastic representation [8]:

$$B_H(t) := \frac{1}{\Gamma(H + 1/2)} \left(\int_{-\infty}^{0} \left[(t-s)^{H-1/2} - (-s)^{H-1/2} \right] dB(s) + \int_{0}^{t} (t-s)^{H-1/2} dB(s) \right), \tag{8}$$

where Γ represents the gamma function $\Gamma(\alpha) := \int_0^\infty x^{\alpha-1} exp(-x) dx$ and $0 < H < 1$ is called the Hurst parameter. The integrator B is a stochastic process, ordinary Brownian motion. The traffic model using Brownian motion is defined: $\overline{A}(t) = (lt + \overline{\sigma} l) B_H(t)$.

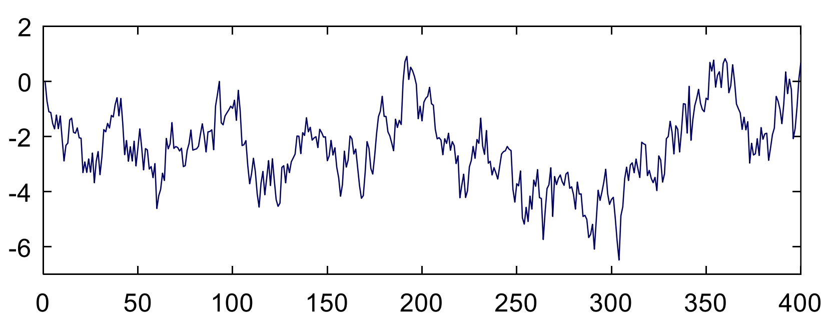

Figure 4. Fractional Brownian motion.

3. Transport problem on a percolation lattice (algebraic structures)

In report shows the possibility of homomorphism stochastic processes in percolation lattice in the context of recognizing the transport properties of these systems. The formal basis for embedding systems is the results of a modern general algebra on the embedding of complex algebraic structures into relatively simple algebraic structures.

In this connection, the principle of fractal homomorphism (universal similarity), in the context of category theory, fixes on the one hand the fundamentality of ***Not What*** is reflected, but ***How***, and on the other hand means the mutuality of fractional structures of any scale [7, 9].

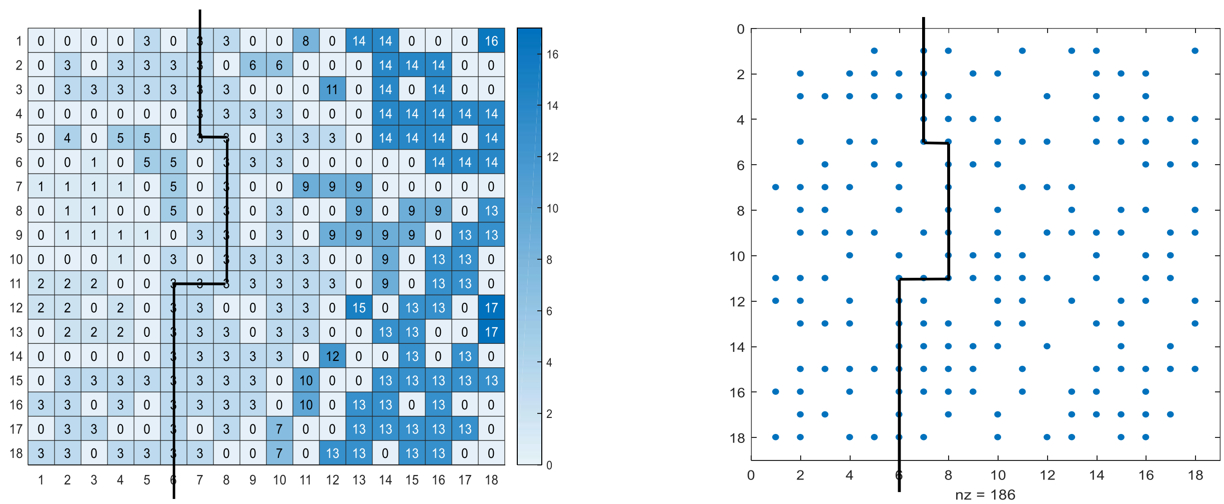

Figure 5. An example of the solution of the transport problem.

3.1 Main provisions

Percolation represents the basic model for a structurally disordered system. The percolation transition is characterized by the geometrical properties of the clusters near p_c. The probability p_∞ that a site belong to the infinite cluster is zero below p_c as [9, 10]:

$$p_\infty \sim (p - p_c)^B . \tag{10}$$

When p approaches p_c, ξ increases as [8, 9]:

$$\xi \sim |p - p_c|^{-\nu} , \tag{11}$$

with the same exponent ν below and above the threshold and ξ - correlation length.

Here p depends on the type of the lattice, the critical exponent and β, and ν they are universal and can be depend only from the dimensions of the lattice.

Axiom of embedding. Let the one – dimensional array 1D $\hat{x} = \{x_i\}_{i=1}^{N}$ be transformed into a square matrix 2d, $\left\| a_{i,j} \right\|$.

The fractal dimension $d_f^{\hat{x}}$ of the analyzed segment of the array is empty.

Then the homomorphism h will be determined as:

$$h : \left\| a_{i,j} \right\| \Rightarrow L \times L, \quad L \times L \in E^2, \tag{12}$$

d_f for reliability.

Then the percolation lattice will represent the geometric and dynamic realization of the stochastic cluster.

3.2 Conductivity of Percolation Lattice

It is noted [11, 12] that the conductivity is represented as:

$$\sigma_{dc} \sim \left(p - p_c\right)^N, \tag{13}$$

where the critical exponent μ is (semi) – universal, $p_c \cong 0{,}592746$ - critical probability.

For percolation on a lattice, μ depends only on d, where d is lattice dimension.

Critical exponent for two lattice dimension equals $\mu = 1.30 \pm 0.002$.

Thus, a transport problem is posed in the context of the homomorphism of stochastic discrete systems onto the percolation lattice.

Conclusion

As a result of the analytical and numerical studies, it can be concluded that it is necessary to take into account a large number of accompanying and influencing parameters of the information network. This approach will allow you to more reliably assess the resources of the existing network and help in choosing the best configuration. Comprehension and application of a large amount of important visual information requires the use of Visual Thinking technology, as well as the use of a set of Deep learning algorithms.

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