Violation of Lorentz invariance as a symmetry

Ikhlov Boris Lazarevitch

Lead Research Engineer

Perm State University

Abstract. A brief classification of symmetry groups is given. It is shown that the existence of M-parity is possible, and the characteristics of the corresponding group are found.

Keywords: renormalization, gauge fields, group characters

Introduction

Cosmological inflation is caused by the transition from a false vacuum to a true one with a spontaneous breaking of the *GUT* symmetry.

Let us list the symmetry groups. Charge parity is the multiplicative quantum number of a truly neutral particle, which determines the behavior of its state vector under charge conjugation. During the charge conjugation operation, the wave function of the particle is multiplied by the *C*-parity value, that is, it changes sign (charge odd particle), or remains the same (charge even). The law of conservation of *C*-parity is fulfilled with strong, electromagnetic, and gravitational interactions, in the case of weak interactions, it is violated. The charge parity of the photon is $C = -1$ (with charge conjugation, the electric charges change sign, therefore, the electromagnetic fields, the quanta of which are photons, must also change sign so that the evolution of the system does not change). Due to the conservation of *C*-parity in any electromagnetic processes, it is impossible to convert an odd number of photons into an even number and vice versa (Farry's theorem). Peony decays due to EM $\pi^0 \to \gamma + \gamma$, $C(\pi^0) = C(2\gamma)$, $C(\pi^0) = C(\gamma)C(\gamma) = (-1)(-1) = +1$.

Combined parity, *CP*-symmetry is the product of *C*-charge conjugation, which transforms a particle into its antiparticle, and *P*-parity, the mirror image of a physical system. The strong and electromagnetic interactions are invariant with respect to the *CP* transformation, this symmetry is broken in the process of some types of weak decay, in the Standard Model - in the quark sector. The *CP*-violation can theoretically be observed in strong interactions, but the *CP*-violating term is here strongly limited by the non-observation in the experiment of the electric dipole moment of the neutron.

Time reversal, *T*-parity is the symmetry of the equations with respect to replacing the time t by $-t$, is written as the equality to zero of the commutator of the Hamiltonian operator and the antiunitary time reversal operator.

Consider the Lorentz transformations, select the z direction. If we complicate the Lorentz group, an imaginary boost with a boost parameter $i\pi$ will tend to $t \to -t; z \to -z$. Let's perform an additional rotation in the xy plane, we get a combination of *P* and *CT*. The combination *CT* appears instead of *T* because the given transformation is unitary, not anti-unitary. Assuming that the operation of complex growth is correct as a symmetry, we get a state that is described by the same laws. Which gives the *CPT*-theorem.

Since the *CP*-symmetry is broken while the *CPT*-symmetry is preserved, it follows that it is not invariable with respect to the *T*-symmetry. The *CPT* theorem states that any Lorentz-invariant local quantum field theory with a Hermitian Hamiltonian must have *CPT* symmetry.
According to the Luders-Pauli theorem, any adequate field theory must be *CPT*-invariant.
The consequence of the *CPT*-theorem is that the particle and antiparticle have exactly equal mass and magnetic moment, their electric charges are equal in magnitude and opposite in sign, spins are equal in magnitude and opposite in direction. In Feynman diagrams, an antiparticle is equivalent to a particle going backward in time, therefore, for example, the Compton effect and annihilation of an electron-positron pair are equivalent and give the same amplitude values.
It is argued that the laws of classical mechanics, classical electrodynamics, quantum mechanics, and the theory of relativity do not change with time inversion.
Physical quantities that change sign with time reversal are *T*-odd, do not change sign - *T*-even. The product of any number of *T*-even values and an even number of *T*-odd values, *T*-even, etc. Coordinate, acceleration, angular acceleration, energy, energy density, force, electric potential, electric field strength, EMF stress tensor, electric charge density are *T*-even. *T*-odd: speed, angular velocity, linear momentum, angular momentum (orbital and spin), power, electromagnetic vector potential, magnetic induction, magnetic field strength, electric current density. Masses, charges and constants not associated with weak interactions are *T*-even.
Accordingly, Newton's second law $ma = \sum_i F_i$,with the addition of rotational motion due to the relationship

$r\omega' = a_\tau$ loses *T*-symmetry. In curved space, energy E and momentum P^i

$$E = \int_{V_3} T^{00}\sqrt{-g}d^3x; P^i = \frac{1}{c}\int_{V_3} T^{i0}\sqrt{-g}d^3x$$

are not saved. That is, there is no *T* and *P* symmetry. One can introduce the pseudotensor of the gravitational field $\partial_k[(T_k^i + t_k^i)(-g)^n] = 0$, n – an integer or half-integer. Then

$$\tilde{E} = \int_{V_3} (T_0^0 + t_0^0)(-g)^n d^3x; \tilde{P}^i = \frac{1}{c}\int_{V_3} (T_0^i + t_0^i)(-g)^n d^3x,$$

However, the new energy and momentum are not complete analogs of energy and momentum in Euclidean space, for example, due to the fact that t^{ik} is not a tensor, the overdetermined momentum is not a vector. In addition, the new energy and momentum are not uniquely determined; they depend on how the pseudotensor of the gravitational field is constructed.

Einstein's equations in the Schwarzschild solution, for example, $-e^{-\lambda}(\frac{\nu'}{r} + \frac{1}{r^2}) + \frac{1}{r^2} = \frac{8\pi G}{c^4}T_1^1$, are asymmetrical

with respect to reflection $r \to -r$. Therefore, in strong gravitational fields, the masses of particles and antiparticles and their charges should not be the same.
Symmetry groups *C, P* and *T* of elementary particles are discrete, groups of gauge transformations and groups in general relativity are continuous. However, these symmetries are related. The Lie algebra by a semisimple Lie group is semisimple, in such algebras there arise systems of roots, the reflections corresponding to them

generate the Weyl group of the given Lie group - this is a certain group of reflections, one of the Coxeter groups.
Breaking the *CPT*-symmetry automatically leads to breaking the Lorentz invariance.
The Lorentz group contains ordinary spatial rotations and reflections, as well as boosts - transformations of the transition from one frame of reference to another.
*O (*1,3) - the orthogonal group of transformations of the Minkowski space that preserve the origin, that is, they are linear operators, consists of homogeneous linear transformations of the coordinates of the four-dimensional space-time $x'_{\nu} = \sum L_{\nu\mu} x^{\mu}$, $x_0 = ct$, $x_1 = x$, $x_2 = y$, $x_3 = z$, leaving invariant form $s^2 = (ct)^2 - x^2 - y^2 - z^2$ and do not change the direction of time. The group includes spatial rotations in three planes, Lorentz transformations, reflections of spatial axes: and all their products. The special Lorentz group *SO* (1.3) is a group of true rotations, a subgroup of transformations, the determinant of the matrix of which is equal to 1 (in the general case, ± 1). The orthochronous Lorentz group $O_{\uparrow}(1,3)$, the special orthochronous Lorentz group $SO_{\uparrow}(1,3)$ — similarly, the only one of the four is connected and isomorphic to the Möbius group, $SO\uparrow(3.1) \subset SO(3.1) \subset O(3.1)$. All irreducible representations of the special orthochronous Lorentz group can be constructed using spinors. The Abelian group of time shifts $Rt \to t + const \in R$ is realized as the group of rotations *S*O(2) of the time plane with coordinates Y1, Y2 (homomorphism onto the group of rotations $R \to SO(2)$). The orthochronous Lorentz group $SO\uparrow(3.1)$ does not change the direction of time, which is expressed by the condition $A^0{}_0 \geq 1$. The time reversal operation for elementary particles without spin consists in changing the sign of *t* and simultaneously replacing the wave function with the complex conjugate in the Schrödinger equation: $\Psi(t,r) \to \Psi^*(-t,r)$. For elementary particles with spin, the time reversal operation consists in replacing: $\Psi_{s\sigma} \to \Psi_{s,-\sigma}(-1)^{s-\sigma}$ [1].
The characteristic of the state of a physical system is the vector of states in the Hilbert space. Time-reversal invariance in the Schrödinger representation means that $\Psi_i \to \Psi_f$ implies $\Psi_f^R \to \Psi_i^R$. The time reversal transformation is specified by the following postulates: $\Psi^R = U^T \Psi^*$, where Ψ - is the state vector of the system, the subscript *T* denotes the transposition operation, and the * sign denotes the complex conjugation operation.
It is postulated that the known symmetries of elementary particles originate from a certain symmetry group *G*. At each phase transition, part of this symmetry is lost, the symmetry group changes: $G \to SU(3) \otimes SU(2) \otimes U(1)$. The group U (1) connected describes electromagnetism, *SU*(2) - the weak interaction, in the framework of the Weinberg - Salam model, combined into an electroweak interaction described by the group $SU(2) \times U(1)$. The strong interaction is described by the *SU*(3) group. Models combining the strong with the electroweak - grand unified theories (*GUT*)
Group *U*(1) – is an abelian continuous group of rotations, local gauge transformations, for the wave function of an electron

$$\Psi'(x) = e^{i\varphi(x)}\Psi(x),$$

which leaves the Lagrangian invariant under the introduction of a compensating vector field identified with the electromagnetic one. Group elements - rotations by arbitrary angles $\varphi(x)$ around the axis. The fermionic doublet in isospace is transformed using the 2-dimensional Hermitian Pauli matrices σ, acting on the spinor:

$$\Psi'(x) = e^{i\vec{a}(x)\vec{\sigma}}\Psi(x) = SU(2)\Psi(x)$$

$\vec{a}$ - are arbitrary real phases. Gauge fields are massless vector W and Y bosons. Accordingly, for the baryon octet - the $SU(3)$ group (not in the space of flavors, but in the space of colors), the compensating fields are colored massless vector bosons - gluons. The gauge transformation is specified by 3 x 3 unitary matrices. Groups $SU(2)$ and $SU(3)$ are also introduced in addition to gauge constructions.

For electromagnetic and strong interactions, *P*, *C* and *T* are symmetry transformations, for weak interactions *P* and *C* are not symmetry transformations, but *T* and *PC* are.

The symmetry groups of Lorentz and other groups in general relativity are not gauge ones.

The idea of gauge gravity was put forward by W. Heisenberg and E. Gapon. There are two gauge symmetries for the gravitational field. The first is given by general covariant transformations of tensor quantities. The field of gauge general-covariant symmetry can be easily identified with the connectedness of the gravitational field (Christoffel symbols). Indeed, expressions for the covariant derivative and gauge connectivity transformations resemble those for the Yang - Mills field. However, it is obvious that deviations from the Minkowski metric cannot be identified with the Goldstone components, an infinite continuous manifold cannot be isomorphic to a finite discrete one. Moreover, there is no analogous expression for the metric tensor. An attempt to describe an arbitrary metric within the Higgs scheme leads only to a transition to the tetrad formalism.

The laws of physics are equivalent with respect to transformations of the Galilean group in Euclidean space, in Minkowski space - with respect to the Lorentz group. But strongly curved spaces are described by other symmetry groups, the Bondi-Metzner-Sachs group, etc. Thus, in a curved space, the *CPT* symmetry should be broken.

Negative mass

One of the corollaries of Wightman's axioms is the *CPT* theorem: there is a general symmetry when parity changes, particle-antiparticle reversal and time reversal, as it turns out, none of these symmetries exist by themselves. That is, the coordinate-time space turns out to be connected with the charge space, although the roots of the groups *C* and *P*, *T* are different, have different dimensions.

In this scheme, there is no symmetry with respect to the change in the sign of the mass.

There are versions of inert mass, active gravitational mass (the source of the gravitational field), and passive gravitational mass. Einstein's principle of equivalence requires that inertial mass be equal to passive gravitational mass, it follows from the law of conservation of momentum that active and passive gravitational masses are equal, that is, all three masses are equal to each other.

The Higgs equations admit the existence of negative inert masses. Negative energy density, that is, negative mass, appears in the Casimir effect.

The possibility of the existence of negative mass was suggested by G. Bondi in 1957 [2]. He built models with different negative masses: inert, active and passive gravitational masses. Bondi and Forward analyzed such

exotic particles [3]. We are talking about negative gravitational mass, so there will be no contradictions with the Higgs models.
Morris et al showed [4] that the Casimir effect can be used to obtain a local region of space-time with negative mass. Forward also proposed a design for a spacecraft engine using a negative mass, which does not require an influx of energy and a working fluid in order to obtain an arbitrarily large possible acceleration [5,6].
At present, particles with a negative effective inert mass in the Bose-Einstein condensate have been obtained [7]. In [8], it was suggested that if the hypothesis of the existence of negative gravitational mass is accepted, the hypothesis of dark matter is not required.
It was shown in [9] that ordinary and dark matter should be continuously generated in the Universe.
The same assumptions were independently expressed in [10]. This paper combines the idea of negative mass with the idea of continuous and homogeneous mass production in the volume of the Universe (which was first proposed in the 1940s). The Friedmann equations are solved with the appropriate additions, but without taking into account dark energy and dark matter, the law of the expansion of the Universe is found. It turned out that the known laws are reproduced if negative mass is produced with a constant velocity $\Gamma = -3H$, where H is the Hubble constant. In this case, the negative mass density will remain constant during expansion, and it will effectively model the cosmological constant. In this case, the expansion rate and lifetime of the Universe are the same as in the ΛCDM model. The evolution of a dense group of particles with positive mass, immersed in a "sea" of particles of negative mass, was also modeled, which should qualitatively describe the evolution of galaxies at the later stages of the expansion of the Universe. We chose $N_+ = 5000$, $N_- = 45000$. As a result, the density distribution was obtained, which is in good agreement with the observational data - the particle density slowly increases when approaching the center of the galaxy and coincides with the Burkert profile. This solves the "sharp halo" problem that arises in the ΛCDM model.
There is a statement that the only criterion for confirming the presence of negative masses can only be astronomical observations. I. Banik discovered a giant ring of galaxies that scatter, as at the moment of the Big Bang, their arrangement is similar to drops from a rotating umbrella, while the rotation of the ring is not observed [11]. Although, ideally, with negative masses, there should be not a ring, but a ball. However, these galaxies fly away from the Milky Way at a much higher speed than that determined by the standard cosmological model.
The spiral galaxy M81 in the constellation Ursa Major, discovered by Art Hoag in 1950 in the constellation Serpent, is also a ring. In the outer part is a ring dominated by bright blue stars, and in the center is a ball of redder stars that are likely much older. Between them is a gap that looks almost completely dark. In the gap between the core and the ring, another ring-shaped galaxy is visible, which is much further away. If it is found that the velocities of the center of the rings and the rings themselves relative to the Milky Way are significantly different, this will testify in favor of the presence of negative masses.
However, the existence of negative masses does not have to be confirmed by the presence of large-scale fragments of matter in space, just as the existence of antiparticles does not have to be confirmed by astronomical observations - in view of baryon asymmetry.

Negative masses also arise during the regularization procedure. The mass of a charged particle must include the energy contained in the particle's electrostatic field. For a sharpened charge, it is infinite: $mc^2 \sim q^2 / r$, i.e. possesses infinite inertia, therefore, the charge cannot move at an accelerated rate. But the total mass of the charge includes the sum of electrical energy and "bare" mass. If the "bare" mass is assumed to be negative, it is possible to renormalize the total mass and obtain the electron mass consistent with the experiment, including with a zero radius.

It is logical, in accordance with the principle of equivalence, to assume the existence of negative gravitational mass. The annihilation of matter with negative gravitational mass and ordinary matter is difficult, since substances with negative mass will not be attracted, but repelled from ordinary matter. At the same time, in ordinary fermions, this negative mass is compensated by a positive electrostatic one. The annihilation of gravitational masses should occur with reactions of the $e^- + e^+ \to 2\gamma$,type, since the electrostatic interaction is much orders of magnitude greater than the gravitational one, and the compensated mass appears in the reaction parameters. However, another option is also possible.

Gravity is 10^{36} times weaker than electrical interaction, 10^{38} times weaker than strong, and 10^{25} times weaker than weak interaction. Let us assume that the decay of particles composed of pairs of particles and antiparticles is analogous to annihilation. Let us assume that the particle lifetime corresponds to the coupling constant.

Pi-mesons consist of quarks and antiquarks of the 1st generation, the lifetime of which is 10^{-6} seconds, it is possible that the annihilation time of gravitational mass and anti-mass is – 10^{-6} x 10^{38} = 10^{32}, which corresponds to the estimate of the proton lifetime.

Let us introduce the operator *M*, which converts the gravitational mass into the antigravitational one:

$$M\Psi(m_g) = \Psi(-m_g)$$

The presence of negative masses is associated with the presence of tachyons, which are possible in the CTO. Consider the expression for the frequency in the formula for the relativistic Doppler effect:

$$\omega = \omega_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

When the speed of light is exceeded, the electromagnetic disturbance ceases to have the form of vibrations propagating to infinity. At $v > c$ the frequency becomes imaginary; therefore, the amplitudes of electromagnetic disturbances will decay exponentially without oscillations: $A \sim e^{-\beta t}$,

where $\beta = \omega_0 \sqrt{v/c - 1} / \sqrt{v/c + 1}$.

That is, the existence of tachyons is not a theoretical error and not a jump beyond the speed limit; tachyons are damped modes. Exponentially growing solutions are not discarded as non-physical, they indicate the instability of the system.

Therefore, it is logical to assume that

$$M\Psi(m_g) = \Psi(im_g); M^2\Psi(-m_g) \quad (1)$$

Accordingly, *M* is a point cyclic group isomorphic to C_4, it can be represented as a combination of rotations in the isospace of the group C_2 with the group of mirror reflections in two planes of symmetry, the fourth Klein group C_{2v} appears, which has 4 classes, respectively, 4 irreducible representations that are one-dimensional, therefore they do not coincide with characters, and energy states cannot be degenerate. The characters of the group are shown in the table:

C_{2v}	E	C_2	v_1	v_2
a_1	1	1	1	1
a_2	1	-1	-1	1
a_3	1	1	-1	-1
a_4	1	-1	1	-1

If the unitary operator of charge conjugation is defined by the equality $C^2 = 1$ and the transformation of a particle into an antiparticle $Ca^+ = b^+$, then the mass conjugation operator is defined through the equalities $M^4 = 1$ and (1).

Full symmetry: $CPT \to CPTM$, so all fermions have negative gravitational mass, their antiparticles are positive, and *CPT*-symmetry is broken.

CPT-symmetry breaking is equivalent to Lorentz breaking, which is believed to occur at high energies, such as in ultra-high-energy cosmic rays. In most models, a dedicated frame of reference is constituted, in which the relic radiation has a Planck spectrum.

In some versions of general relativity, the Lorentz invariance is violated, the relation $E = p^2 + m^2$ is replaced by a relation. $E = p^2 + m^2 + p^4 / z; z \sim m_{Plank} \sim 10^{19} GeV$ [12]. The corresponding Hamiltonian of a free particle - $H = \sqrt{p^2 + m^2 + p^4 / z^2}$, hence the Lagrangian $L = -m\sqrt{1 - v^2 / c^2} - \frac{p^4}{2z^2 m}(1 - v^2 / c^2)^{-3/2}$. Other forms of the Hamiltonian are considered, as well as a model with an additional dimension, in which spontaneous violation of Lorentz invariance occurs due to the condensate of vector fields [13]. In the Gross-Neveu model (with the Lagrangian of the Extended Standard Model, SME), an additional vector b is introduced, which violates Lorentz invariance [14]:

$S(\bar{\Psi}, \Psi) = \int d^3x [\bar{\Psi}\gamma_j(\partial_j - ib_j)\Psi + \bar{\Psi}m\Psi - \frac{G}{2N}(\bar{\Psi}\Psi)^2]$. In the theory of Khorzhava, in the modification of general relativity, an additional scalar field, a chronon field, with an excess of the speed of light, etc. is activated.

These constructions seem to be artificial, since in the Riemannian metric the particles move outside the light cone of Minkowski space without the presence of additional terms in the Lagrangian.

Violation of Lorentzi invariance means violation of *CPT*-symmetry, i.e. violation of parity of mass and inequality of masses and charges of particles and antiparticles. *CPTM*-symmetry is also not preserved in these models.

Reflection groups are Coxeter groups [15]. In the one-dimensional case, the product of reflections on the Euclidean plane *t* and *x*, the mirrors of which intersect at the origin, forming an angle π / m, identical to rotation around the origin by an angle $2\pi / m$,i.e. on the 180°. The group $PT = (t, x)$ satisfies the relations $t^2 = x^2 = 1$.

The roots of the groups *P* and *T* are vectors, *P, T* are groups over a field of vectors in the Minkowski space, *C* - is a group over a discrete field of real numbers, where the addition operation is given.

PT and *C* in the coordinate-charge space form the C_{2v} group, that is, the characters of the *CPT* and *M* groups coincide. Accordingly, the *CPTM* transformation, like two successive *CPT* transformations, preserves the sign of the bosons and reverses the sign of fermions.

The threshold for the production of $E = 2m_g c^2$ pairs is lower than the threshold for the production of electron-positron pairs, since along with the creation of an electron-positron pair, gravitational masses with opposite signs are also born.

By virtue of Noether's theorem, the groups *C, P, CP*, and *T* correspond to parity conservation laws (time is isotropic); therefore, *M*-parity must also exist. Accordingly, *MT*-symmetry must also exist.

The continuous mass shift group is realized similarly to the standard Lie group of time shifts, $m_s = m + s,\ s \in \mathbb{R}$.

Mass ceases to be a parameter and becomes a function argument.

Conclusion

The question whether it is possible to construct the group *G* as including the *M*-symmetry remains open, since group *M* is not a calibration group.

If the negative mass hypothesis is valid, it could become a competing dark energy hypothesis.

It can be assumed that the asymmetry between particles and antiparticles is of the same nature as the asymmetry of both masses and anti-masses, and these asymmetries, together with the direction of time and the absence of Dirac's monopole, determine each other.

From which it follows that the stated hypothesis of the existence of negative masses is not related to the Hoyle, Bondi and Gold model of continuous generation of matter in the Universe (at the same time, this generation is possible if we consider the thermodynamics of the expanding Universe [16]).

This moment distinguishes the stated hypothesis from the version of Terletskiy [17] with the creation of quadrigues of particles from the vacuum (positons and negatons). Terletsky does not use the apparatus of group theory, his justification for the impossibility of detecting negatons is doubtful, in addition, in this article, by analogy with an electric charge, we are talking about a negative gravitational mass, but not inert.

It is also worth emphasizing that the negative energy density arising in some Casimir effects is associated with the vacuum states of the fields, but not with the negative mass of the particles.

References

1. Landau L.D., Lifshitz E.M., Quantum mechanics. - M., Science, 1963. - P. 249

2. Bondi H. «Negative Mass in General Relativity», Rev. Mod. Phys. *29* No. 3 July 1957, P. 423ff

3. Landis G. Comments on Negative Mass Propulsion. J. Propulsion and Power, Vol. 7 No. 2, 304 (1991).

4. Morris M., Thorne K., and Yurtsever U. Wormholes, Time Machines, and the Weak Energy Condition. Physical Review, 61, 13, September 1988, P. 1446—1449

5. Marc G. Millis, "The Challenge To Create The Space Drive", 1997, Journal of Propulsion and Power.

6. Marc G. Millis & Eric W. Davis (editors): Frontiers of Propulsion Science. American Inst. of Aeronautics and Astronautics, Reston VA, 2009, ISBN 978-1-56347-956-4 (739 pages), Summary

https://en.wikipedia.org/wiki/Breakthrough_Propulsion_Physics_Program#Diametric_drive
7. Khamehchi M. A., Hossain K., Mossman M. E. Zhang Y., Busch Th., Forbes M. M, Engels P. Negative mass hydrodynamics in a Spin-Orbit--Coupled Bose-Einstein Condensate. Phys. Rev. Lett. 118, 155301 (2017)
8. Ikhlov B. L. Life in the Universe. Scientific Research of the SCO Countries. Part 1. China, 2018. December, 12. P. 236-245.
9. Ikhlov B.L. Thermodynamic approach in cosmology. Eurasian scientific journal. 2019. №1. http://journalpro.ru/articles/termodinamicheskiy-podkhod-v-kosmologii/?sphrase_id=14205
10. Farnes J. S. A unifying theory of dark energy and dark matter: Negative masses and matter creation within a modified ΛCDM framework. Astronomy & Astrophysics. 2018. V. 620. №A92. DOI https://doi.org/10.1051/0004-6361/201832898
11. Banik I., Zhao H. Plane of high-velocity galaxies in the entire Local Group. 21.1.2017. Astrophysics of galaxies. Monthly Notices of the Royal Astronomical Society. Cornell University Library. arXiv.org > astro-ph > arXiv: https://translate.googleusercontent.com/translate_c?depth=1&hl=ru&prev=search&rurl=translate.google.ru&sl=en&sp=nmt4&u=https://arxiv.org/abs/1701.06559v4&usg=ALkJrhhe68QEjZNHf1y8aVTxKB3BL-fwAA
12. Rubtsov G., Satunin P., Sibiryakov S. On calculation of cross sections in Lorentz violating theories. arXiv:1204.5782. See also: Plotnikova D. V. Lorentz invariance violation and bound states. MSU, 2015.
13. Libanov MV Particle physics and cosmology in models with additional spatial dimensions and Lorentz invariance violation. Abstr. diss. cand. phys-mat. sci. M., RAS INR, 2009.
14. Gubina N.V., Zhukovskii V. Ch., Kurbanov SG, Dimensional reduction of fermions in the Gross – Neveu model under conditions of broken Lorentz invariance. Moscow University Bulletin. Series 3. Physics. Astronomy. 2012. № 2.
15. Hamphreys J. E. Reflection groups and Coxeter groups. Cambridge Universoty Press, 1990.
16. Ikhlov B.L. Thermodynamic approach in cosmology. Eurasian scientific journal. 2019. №1. http://journalpro.ru/articles/termodinamicheskiy-podkhod-v-kosmologii/?sphrase_id=14205
17. Terletskiy Ya. P. Cosmological consequences of the hypothesis of the creation from vacuum of complexes of particles of positive and negative mass. On Sat. "Modern theoretical and experimental problems of the theory of relativity and gravitation", materials of the VII All-Union conference. Yerevan, 1988. See also: Terletskiy Ya. P. Paradoxes of the theory of relativity. M., Science, 1966.