Stochastic Modelling for Evolution of Globular Star Cluster Omega Centaury

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Abstract. The preliminary stochastic model for the evolution of the globular star cluster NGC 5139 is presented, including technique to simulate ecosystems associated with parent stars based on terrestrial exoplanets. The approach is based on the Monte Carlo method using generation of the rate of star formation in discrete time intervals in accordance with the available data on the prevalence of spectral classes of stars within the modeling area. A mechanism for recursive correction of the star formation rates based on the comparison of the model structure with astronomical data has been developed. The equation of convolution functions for birth and death of stars with ecosystems passing through cosmological filters in connection with a composition of the nearest stars in the tetrahedral network is introduced. The proposed stochastic model can be used to evaluate the transmissions between technospheres in habitable zones of cluster. The transportation or signal propagation along the edges of the tetramesh is an alternative method compared to the Landis approach based on percolation theory for the numerical solution of the Fermi paradox. The Delaunay 4D mesh and Voronoi's mosaic can be used to outline the location hulls of cluster regions with biosignatures where technospheres are likely in state of communicative phases and may be detected by SETI programs.

Keywords: stochastic model, globular cluster, star evolution, convolution equation, spectral class, Delaunay tetrahedralization, Voronoi mosaic, SETI, biosignature.

Introduction

Star Cluster NGC 5139 Omega Centauri is the largest in our Galaxy, the brightest and most massive globular cluster known. It is 15000 ly distant from Earth, making it one of the closest clusters. The cluster is about 150 ly across and includes about 10^7stars, the mass of the cluster is $5 \cdot 10^6 M_{\odot}$. The density near the center of the cluster is about $2 \cdot 10^3 M_{\odot}/pc^3$. Most of the stars are main sequence stars in the Hertzsprung-Russell diagram. There are a number of red giants - stars in the final stages of evolution. Recently radio pulsars have been found in the center of the cluster.

To evaluate planetary products and astrobiological aspects of evolution with the well-known energy classification of cosmic civilizations by Kardashev H.C. [2] we are using also the next classification based on communicative opportunities of possibile intelligent life signatures inside the boarders of their planet Ns – noosphere, in the star planetosphere Ts – technosphere, in interstellar space inside the volume of the Galaxy Gs - galosphere and in the metagalaxy or in the universe Ms - metasphere. These characteristics are also taken into account as probabilistic parameters in the model of cluster evolution based on the stochastic approach using the Monte Carlo method, Markov chains [1], and the equation for the convolution of the birth and death functions of stars.

Purpose of the study – to establish the stochastic evolutionary model of globular star cluster NGC 5139 in connection with the star formation rates and spectral composition of the population throughout the lifetime of the cluster. One of the favorable features of the Omega Centauri cluster is that the stars in it consist of main-sequence spectral types and have different ages and metallicities, while in most clusters they form almost simultaneously and practically do not differ in chemical composition. The stars in the cluster are generally up to 10 Gyr in age, that is, they were not formed uniformly but during at least two bursts of star formation. Taking into account the peculiarities of the object Fig. 1 shows a block diagram of stochastic modeling.

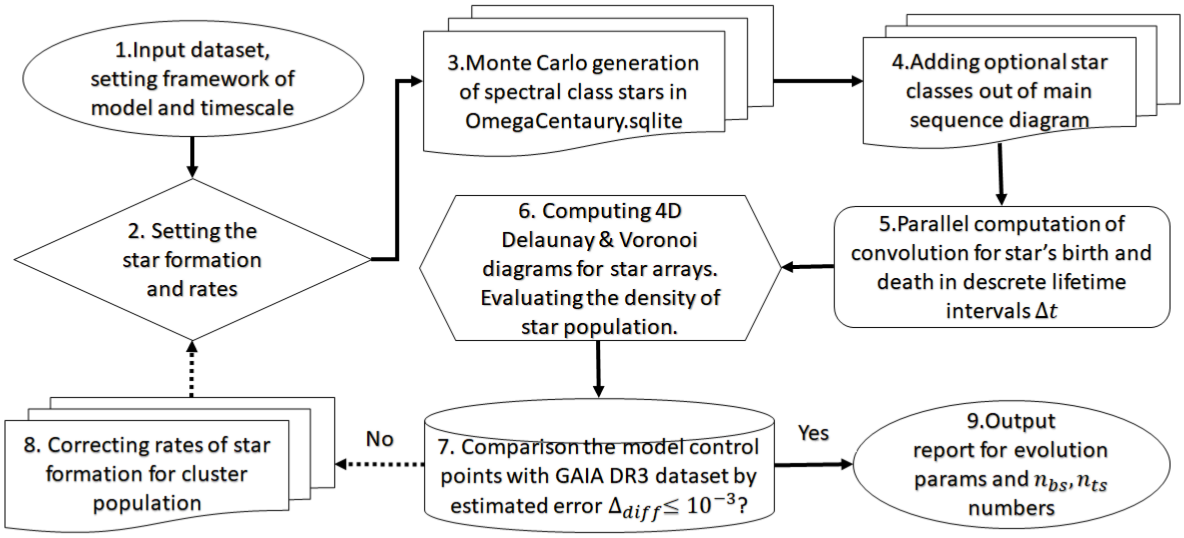

Fig.1. Block diagram of stochastic evolution modeling star cluster NGC 5139

An important point is setting the rate of star formation, SFR. If the Milky Way is currently forming up to $10M_{\odot}$ masses of the Sun per year and 10^7 per 0.1 Gyr, then in the globular cluster NGC 5139, the activity of star formation is about 5 orders of magnitude less intense. As a first approximation, the model uses a uniform distribution over time intervals. Stellar dynamics based on taking into account gravitational forces using the N-body method [4] and the kinetics of fragmentation of gas clouds during compaction of matter can be included in the model to increase the detail of the evolution of the cluster at all stages of its formation.

Materials and methods

The GAIA mission data release (DR3) provides the positions and mean proper motions for $> 10^9$ and more then 150 Milky Way globular clusters with a typical uncertainty of 0.05 mas yr -1 limited systematic errors. We use also some variants of Monte Carlo Markov Chain (MCMC) technique and common parameters from [5]. Table 1 shows the initial initialization settings for the stochastic model.

Spectral star class	Star formation rate n_s / 0.1 Gyr	Fraction, units	Lifetime, Gyr
O	10^3	0.00001	0.01
B	$2 \cdot 10^3$	0.00109	0.10
A	$5 \cdot 10^3$	0.0519	1.00
F	10^4	0.037	3.50
G	$2 \cdot 10^4$	0.06	10.0
K	$4 \cdot 10^4$	0.12	50.0
M	$6 \cdot 10^4$	0.73	200.0

Table 1. Initialization parameters of the stochastic model

The algorithm for simulating the evolution of the globular cluster includes:

Step 1. Setting the number of stars in a representative sample of input data $N_s = 10^7$. The modeling area is a sphere with a diameter of 150 light years with a given distribution of stars, chosen depending on the generation method. To record and store dataset the OmegaCentaury.sqlite tables of SQLite data base is implemented.

Step 2. The coordinates within the framework of the model and the spectral class of the main sequence stars are set using the Monte Carlo method. Non-main sequence star types for classes W, L, T, Y, C, S and D may be generated optionally. The lifetime of stars is installed in relative units within each spectral class. Cluster age is installed in the range from 0 to 10 Gyr. In the population, 20% of stars are considered binaries and triples in different proportion.

Step 3. After the end of the life span of stars in classes O, B, A, F, G, new stars of one of the spectral types are generated in a random position or, in the case of the third generation, at a distance of one light year from the previous position to maintain a constant number and density in generations of stars. The processes of the formation of new stars from gas-dust clouds and stellar dynamics in the model are partially taken in account by according to the data of their own velocities. The probabilities of transition from blue giants to red supergiants or from blue giants to red giants and so on with a relatively short lifetime can be taken into account as optional parameters.

Step 4. Stars of all classes except O, upon reaching the age $T_s \geq 0.1\, Gyr$ form planetospheres with the probability $P_{ps} \sim 1$, stars of all classes, except for O and B, with an age exceeding $T_s \geq 1\, Gyr$ can have planets with lithospheres (with an earth-similarity index $PHI \geq 0.7$) with a probability $P_{ls} = 0.4$.

Step 5. Stars of classes F, G, K, M at age $T_s \geq 3\ Gyr$ can form earth-like planets with biospheres with probability $P_{bs} = 0.3$. The lifetime of stars of the K and M classes is much longer than the entire age of the cluster.

Step 6. Stars of classes G, K, M with age exceeding T_s≥4 Gyr can have earth-like planets with noospheres with probability P_ns = 0.2. , and stars of classes G, K, M with age exceeding T_s≥4.5 Gyr with probability P_ns = 0.1 can have Earth-like exoplanets with technospheres

Step 7. The marking of stars with lithospheres, biospheres, noospheres and technospheres is performed in the database tables in accordance with the established probabilities.

Step 8. Construction of Delaunay grid diagrams and Voronoi mosaics to compare options for a) isotropic transmission of communication signals and b) transmission of SETI messages with signal amplifiers and repeaters located at the nodes of the tetrahedral network.

Step 9. Calculation of integral values according to the discrete equation of convolution of the birth and death functions of the population of stars and statistical indicators of the expansion of the search to the peripheral regions of the cluster.

When constructing grid models of DT / VD stellar arrays, Delaunay tetrahedralization and Voronoi diagram, the TetGen grid generator is used [10]. In the calculation of the local cosmic filter of evolution φ (s, t), the data on the nearest neighbors of each star, their spectral classes, are determined by the tetrahedral Delaunay network. After comparing the model with the actual data on the spectral classes of the cluster, there is also a change in the Markov chain of transition probabilities and a restructuring of the star formation rate over discrete time periods.

Based on the available data that the composition of the stars of the observed cluster NGC 5139 is close to the ones of the stars of the main sequence of the Milky Way, it is possible to estimate the probability of the formation of planetospheres and exoplanets in the habitable zones of stars. Random processes η (t) are taken into account by specifying the values of the Markov chain in the form of star formation intensity rates for the evolution intervals. The resulting convolution of the functions η (s, t) of creation and death of stars for the discrete case was taken in the following form

$$\eta(s,t) = g \otimes f = \sum_{\lambda=\lambda_0}^{S} \sum_{\tau=\tau_0}^{T} g(\lambda,\tau) \cdot (\psi(\lambda,\tau) \cdot \varphi(s-\lambda, t-\tau)) \tag{1}$$

where the summation is performed over spectral classes S with characteristic radiation wavelength λ and over time T intervals τ within each time interval. The population density is estimated. Thus for each spectral star class Markov chains of transition probabilities for the stages with planetospheres to ones with metaspheres are specified, as shown in Table 2.

j \ i	O	B	A	F	G	K	M

P_s	p^O_{Ps}	p^B_{Ps}	p^A_{Ps}	p^F_{Ps}	p^G_{Ps}	p^K_{Ps}	p^M_{Ps}
L_s	p^O_{Ls}	p^B_{Ls}	p^A_{Ls}	p^F_{Ls}	p^G_{Ls}	p^K_{Ls}	p^M_{Ls}
H_s	$\otimes$	p^B_{Hs}	p^A_{Hs}	p^F_{Hs}	p^G_{Hs}	p^K_{Hs}	p^M_{Hs}
B_s	-	$\otimes$	p^A_{Bs}	p^F_{Bs}	p^G_{Bs}	p^K_{Bs}	p^M_{Bs}
N_s	-	-	$\otimes$	p^F_{Ns}	p^G_{Ns}	p^K_{Ns}	p^M_{Ns}
T_s	-	-	-	$\otimes$	p^G_{Ts}	p^K_{Ts}	p^M_{Ts}
G_s	-	-	-	-	$\otimes$	p^K_{Gs}	p^M_{Gs}
M_s	-	-	-	-	-	$\otimes$	p^M_{Ms}

Table 2. The matrix of markov chains for transition probabilities between stages.

In the table 2 there are O, B, A, F, G, K, M – spectral classes of stars; $P_s, L_s, H_s, B_s, N_s, T_s, G_s, M_s$ – stages of stars with namely planetospheres, lithospheres, hydrospheres, biospheres, noospheres, technospheres, galosphere and metaspheres correspondingly; p^A_{Bc} - transition probabilities between one stage and another. $\otimes$ - the cell with Great Filter of life evolution. An illustration of the Markov chain in the form of graph for evolutionary stages for stars of spectral class M is shown on Fig. 2.

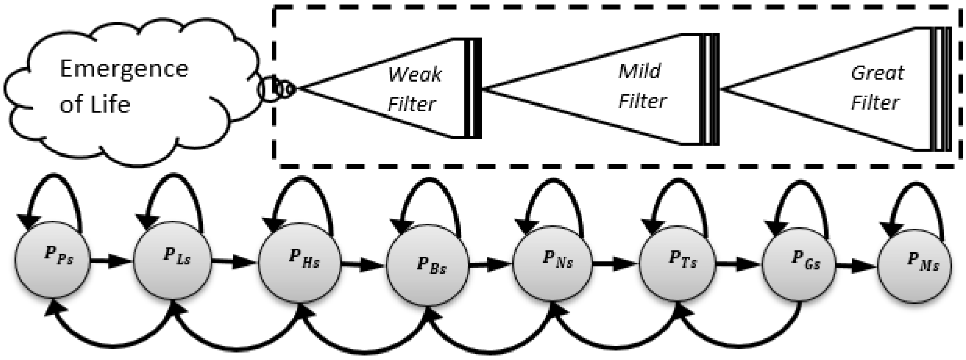

Fig.2. The graph of Markov chain for transition probabilities between stages of parent stars and filters of life evolution

The nearest neighbors are determined by the tetrahedral network of the Delaunay diagram. Expansion is possible to stars of classes F, G, K, M if they have formed planets with lithospheres. Short-lived blue giants, pulsars, magnetars, red giants like Aldebaran, and supergiants like Betelgeuse are not suitable for inclusion in the sphere of influence. In contrast to the Landis percolation model [6], the colonization process is modeled using the Delaunay tetrahedral stellar grid, rather than a two-dimensional grid of cells.

To simulate expansion, transport and colonization, the probability of the expansion of technospheres and metaspheres to neighboring stars is set with the conditional probability P_ex = 0.1 per unit time. The movements are limited by the speed of light and can take place along the optimal paths along the edges of the tetrahedral grid for bypassing areas with dangerous objects of the Galaxy. When the N-body stellar dynamics mechanism is included in the model [4], the effect of the global ψ (λ, τ) and local φ (λ, τ) filters of the φ (s, t) evolution can be enhanced by increasing the probability of collisions of stars and ejection from the system or destabilization of planetospheres in the central regions of the cluster.

The habitable zone of the SHZ cluster is not assessed as such, but for all stars the average distance between them should be at least $D_{pair} \geq 0.1ly$, otherwise, the formation of planetospheres around this pair of stars is an unlikely event. Delaunay tetrahedral networks and polyhedral networks of Voronoi mosaics are constructed using the TetGen mesh generator (https://wias-berlin.de/software/tetgen/index.html) as a whole for the entire set of stars, including stars outside the main sequence of the Hertzsprung-Russell diagram and separately for each spectral class. It is assumed that if a higher probability of the formation of technospheres in K-class stars is revealed, the network of communications and transport will develop mainly among stars of this type.

The visualization was performed using the GLScene graphic engine (https://github.com/GLScene/GLScene) with Fermi Paradox Simulator software, where different cadencer components were used to process the discrete age intervals for each spectral class of stars. The color palette for the visual spectral types corresponded to the colors of stars of the Harvard scale to allow comparison of the resulting images of the model and astronomical images obtained with satellite telescopes. Figure 2 shows images of the cluster and the Delaunay tetrahedral network of its central part.

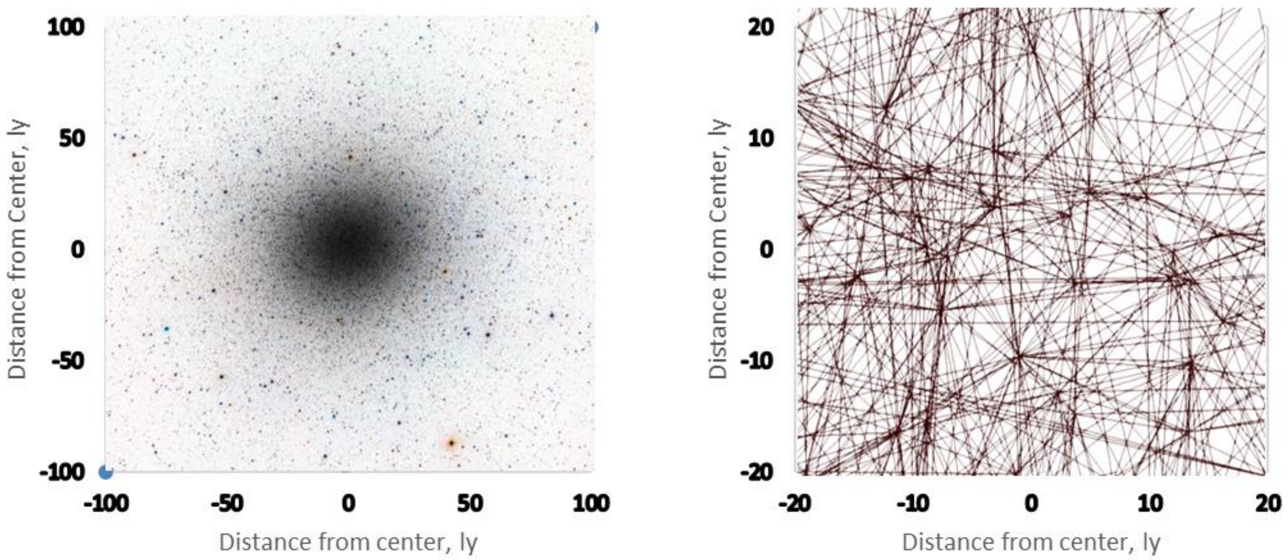

Fig.2. The globular star cluster Omega Centauri NGC 5139 (left, credit ESO) and Delaunay tetrahedral mesh for G-class stars near central core (right)

In the center of the cluster, the distance between the stars is about 0.2 ly. When visualizing the evolution of the model in a time period from 0 to 10 Gyr, discrete time intervals of the timer are selected in the range from a millisecond to an hour. Color of glow sprites for stars with clOlive biospheres, clGreen noospheres and clLime technospheres.

The stellar evolution is spawned and closely related to the likelihood of the formation of planetospheres with Earth-like exoplanets. In this regard, to reflect the connection with the spectral types of stars, it is proposed to use the Drake equation [3] in the following modification:

$$n_{ts} = I_{ts} + (N_s - I_{ts}) \cdot f_{GHC} \cdot \sum_{k=0}^{M} \left(P_k^{gr} \cdot P_k^{fl}\right) \cdot \left(p_k^{ps} \cdot p_k^{ls} \cdot p_k^{bs} \cdot p_k^{ns} \cdot p_k^{ts}\right) \quad (2)$$

where n_{ts} – number of stars with technospheres; I_{ts} – number of technospheres actually known, currently $I_{ts} = 1$; N_s - the number of stars in the cluster is estimated as 10^7; f_{GHC} – fraction of stars in a safe zone or outside close proximity; P_k^{gr} - the abundance of stars of the spectral type; $p_k^{ps}, \ldots, p_k^{ts}$ - the probabilities of the formation of stars of various classes of ecospheres on terrestrial planets with the index $PHL > 0.9$; e.g. $p_k^{bs} = \frac{T_k^s - T_{sb}}{T_k^s}$ - the probability of the formation of biospheres, T_k^s – the average stellar class lifetime, T_{sb} – the average time to start bioevolution $\geq 3\ Gyr$; p_k^{ns}, p_k^{ts} - the likelihood of the noospheres and technospheres formations.

Results and discussion

The Monte Carlo method with markov chain matrices and descrete convolution equations for star's birth and death in different spectral classes the new treatment to solving Fermi paradox was implemented. Parallel computational software project SFPS (Stochastic Fermi Paradox Simulator) has included two evolutional filters for the dynamic convolution as a global cosmological strong $\Psi(s,t)$ and a local planetary weak $\Phi(s,t)$. Markov triangular matrix of transition probabilities in conjunction with $N - body$ and relaxation method could be used for procedural generation of planetary worlds which connected with appropriate spectral star classes. The L / T coefficient of the probability to find a technosphere in the communicative phase, that is, the ratio of the length of the message transmission period to the age of the cluster, is not used, since it is not the total number of technospheres in history that is determined, but their possible number at the current moment. The self-destruction of technospheres or the scenario of space wars are not considered; therefore, the upper optimistic threshold for the growth of the number of semi-potential signal sources is estimated. Figure 3 shows the growth curves of the population of stars at a fixed rate of star formation and the growth curves of ecosystems in the form of biospheres and technospheres associated with parent stars.

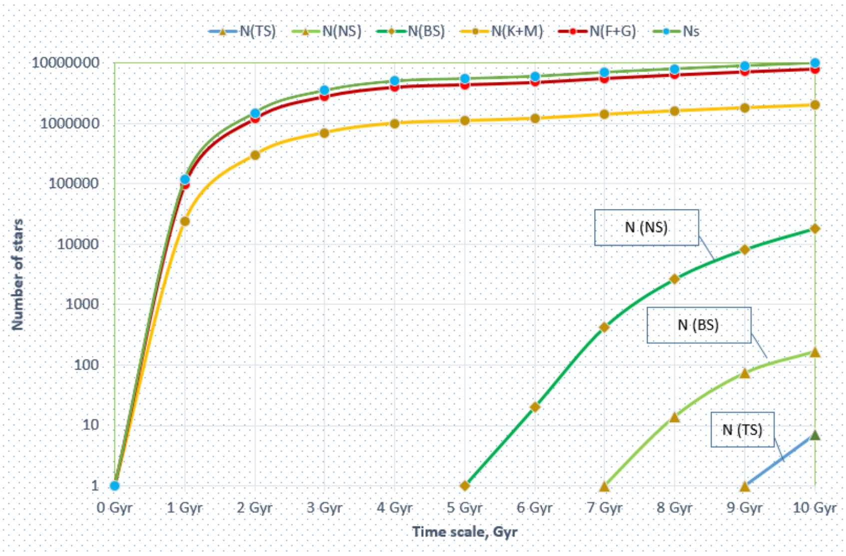

Fig.3. The star formation rates and associated
potential numbers of bio- noo- and technospheres

To characterize the communication potential of a cluster for SETI, one can use the volumetric W_{vt} coefficient of the prevalence of technospheres in the signal transmission phase. The estimate of the potential W_{vt} – is the ratio of the volume of the cells of the Voronoi mosaic v_{ts} , with active technospheres to the total volume of the network of polyhedra v_s в диаграмме Вороного скопления $NGC\ 5139$. in the Voronoi diagram of the NGC 5139 cluster. Thus, the communication potential to find technospheres in globular cluster Omega Centauri, according to the optimistic scenario is $W_{vt}{\sim}10^{-5}$ in the current time interval τ for $N_s{\sim}10^7$ stars.

Based on the simulation results, the stochastic model gives an estimate of the number of biospheres in a cluster of almost 18200, while the number of noospheres does not exceed 170 units, and technospheres, presumably, about 7. But if we take into account the possibility of the formation of terrestrial exoplanets in binary and triple systems, then these figures can be higher. If we assume that the radius of reliable registration of bio- and technosignatures by spectroscopic methods is no more than 20 pc, and the range of isotropic signal propagation is not over 50 pc, then it is possible that technospheres of the cluster have established contacts with each other for information exchange.

Conclusion

In the paper a stochastic approach to simulate star cluster evolution on the base of NGC Omega Centauri is presented. A new approach is outlined to follow the known star locations together with stochastic procedural generations of main-sequence stars for the cluster from the beginning of evolution by a Monte Carlo technique. An advanced convolution function for the birth and death of stars was used in descrete form in accordance with Markov chain matrix of transitive probabilities. The Delaunay 4D tetrahedral meshes and Voronoi mosaics are required to take in account nearest star neighbours and find distances for isotropic transport propagations or network communications with space restrictions. The parallel computational method can be further extended to generate an efficient and realistic models for large star clusters. This results of the simulation of star evolution in the local Solar neighbourhood [9] coud be applied also as alternative to Landis percolation approach to quantify the Fermi paradox. In the article [10] is argued that the factor L as lifetime of communicative civilization in Drake formula [3] is in fact the most important regarding the practical implications SETI, because it determines the maximal extent of the "sphere of influence" of any technological civilization or technospheres in our names. In the described above stochastic model only lifetime L of technospheres is limited by the age of stars of the corresponding spectral classes but not associated and more advanced entities.

In future work the simulator can be verified with a more accurate adjustment of astrobiological parameters and values for the probabilities of the formation of terrestrial exoplanets, both earthlike and superearth ones based upon data processing from ground-based telescopes and new surveys from the mission of GAIA satellite telescope [5]. The probabilistic evaluations for the rates of evolution for biospheres, noospheres and technospheres in more then 150 globular clusters of our Galaxy can represent an additional justification in solving the Fermi paradox [7, 8] by numerical methods, taking into account the metallicity and star types of various spectral classes.

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